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MINIMIZAÇÃO DO DESGASTE NAS CURVAS COM RAMPAS RETAS EM FERROVIAS

MINIMIZATION OF WEAR DOWN OF RAILWAY'S CURVES WITH STRAIGHT RAMPS

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Leon Sinay**

Railway's lay-outs are, in general, formed by straight lines, matched by circular curves, and transition segments consisting of variying curvature curves. The goal of these transition segments is to avoid bumps that, otherwise, would occur between the weel's brim and the inner face of the rail's head as trains pass from the straight to the circular part.

When a body moves along a curved path, it is subjected to a centrifugal acceleration, which depends on the body's velocity. This acceleration can be counterbalanced by a lateral slant of the path which, in railways, is obtained by rising the outer rail.

Since, in practice, several kinds of trains use the same railway, the lifting of the rail which is adequade for one kind may not be for others, and therefore, wearing-down of the rail is not avoided.

The purpose of this paper is to show that wear-down of the outer rail of curves with straight ramps can be minimized using Operation Research techniques. The wear-down problem is equated from physical laws, and the wear-down is minimized using Fibonacci's method, programmed for a microcomputer.

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1. INTRODUCTION

Railway's lay-outs are formed by straight lines matched by curved arcs. Due to its simplicity, circular curves are being used to design this matching arcs since long ago. Fig. 1 shows two straight lines AB and CD with intersection at P, matched by a circular curve of radius R. In the technical language of road engineering, the lines AB and CD are called tangents.

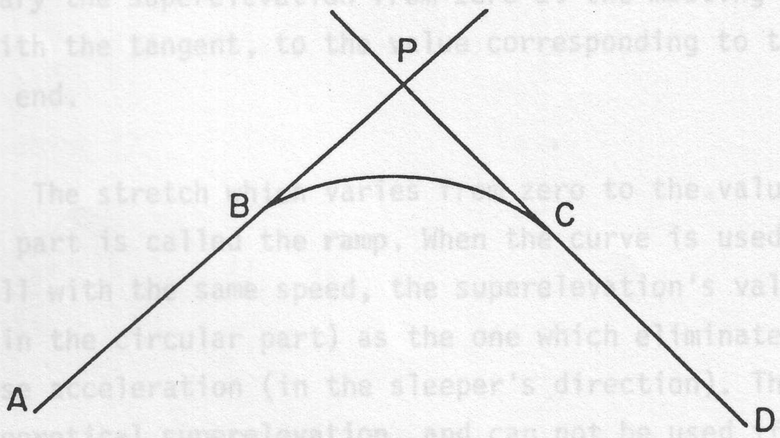


Fig. 1

Matching by circular curves presents two disadvantages: the dificulty in distributing smoothly the rise of the outer rail, and the bump between the wheels' brims and the inner face of the rail's head, which takes place at the entrance and exit of curves, and due to the fact that the change from the straight line to the circular curve is so sudden that it does not give time to the bogey to reach its right position. It is then necessary to include in the lay-out a transition stretch which makes the matching smooth. This is done introducing a varying curve between the straight and circular parts of the railway. Two typical transition curves are cornu's spiral and the cubic parabola. Brazilian federal laws for railways do not make the use of either one mandatory, however, according to Pacheco de Carvalho [1], it is used the spiral rather than the parabola in the few railways with transition in the country.

Going through a curve, the wagon is subjected to gravity and the centrifugal acceleration (which, it is well known, depends on the train's velocity and curve's curvature). The component of the sum of both, in the direction of the sleepers, can be anhilated by the leaning of the wagon due to rise of the outer rail with respect to the inner one. Such a difference in the levels, called superelevation, must obey certain limits, fixed according with the speeds of the trains that travel on the railway.

In the circular part of the railway, the norm of the acceleration is constant (considering the train's speed constant), therefore, the superelevation must also be constant and obviously positive. Since in the straight part of the railway the superelevation is zero, the transition curve is used to vary the superelevation from zero at the meeting point of the transition with the tangent, to the value corresponding to the circular section at the other end.

The stretch which varies from zero to the value corresponding to the circular part is called the **ramp**. When the curve is used for trains that can travel all with the same speed, the superelevation's value can be determined (in the circular part) as the one which eliminates the effects of the transverse acceleration (in the sleeper's direction). This superelevation is called **theoretical superelevation**, and can not be used if the railway is utilized for trains with different speeds. If, for example, the theoretical superelevation is determined using the fastest train, then, the slower trains suffer an acceleration toward the interior of the curve, over-wearing the inner rail. According to Coelho [2], the curve's wear-down is the main reason for substitution of rails in all world's railways. This paper's goal is to show a method which permits to determine the superelevation which gives a minimum wear-down in the wheels and rails in the whole curve (ramps and circular sections).

2. CORNU'S SPIRAL PROPERTIES

Let us consider the particular case in which the horizontal projection of the ramp is the transition curve. If this is a cornu's spiral and at each point the slope is proportional to the distance from the ramp's origin up to the point, then the ramp can be mathematically described in the parametric form by

$$X(t) = (x(t), y(t), z(t)) \quad (1)$$

with

$$x(t) = \frac{a}{\sqrt{2}} \int_0^t \frac{\cos \tilde{t} \ d\tilde{t}}{\sqrt{\tilde{t}}} \ , \ y(t) = \frac{a}{\sqrt{2}} \int_0^t \frac{\sin \tilde{t}}{\sqrt{\tilde{t}}} \ d\tilde{t}; \qquad (2)$$

$$z(s(t)) = m.s(t) \qquad (3)$$

where a and m are adequated constants and s is the curve's arc-length. The arc-length's differential is given by

$$ds = ((x')^2 + (y')2 + (z')^2)^{1/2} \ dt \quad ()' = d/dt,$$

and therefore, using (2),

$$s = \int_0^t \left(\frac{a^2}{2\tilde{t}} + (z')^2 \right)^{1/2} d\tilde{t}. \qquad (4)$$

Substituting s in (3) by (4) and differentiating with respect to t, we have

$$(z')^2 = \frac{m^2 a^2}{2(1-m^2)t}$$

from which it follows that

$$z(t) = \frac{\sqrt{2}\ ma}{\sqrt{1-m^2}} \sqrt{t}$$

and

$$s(t) = \frac{\sqrt{2}\ a}{\sqrt{1-m^2}} \sqrt{t} \qquad (5)$$

We can express (1) as a function of the arc length s, using (5) results in

$$X(s) = \left(\frac{a}{\sqrt{2}} \int_0^{t(s)} \frac{\cos \tilde{t}}{\sqrt{\tilde{t}}} \, d\tilde{t}, \ \frac{a}{\sqrt{2}} \int_0^{t(s)} \frac{\sin \tilde{t}}{\sqrt{\tilde{t}}}, \ ms \right) =$$

$$= \left(\sqrt{2} \ ac \int_0^{s} \cos(c\tilde{t})^2 \, d\tilde{t}, \ \sqrt{2} \ ac \int_0^{s} \sin(c\tilde{t})^2 \, d\tilde{t}, \ ms \right) \tag{6}$$

where

$$c = \frac{\sqrt{1-m^2}}{\sqrt{2}\, a} .$$

When the train's speed is constant alon the ramp, the vector d^2X/ds^2 is in the plane perpendicular to the curve's tangent vector. It follows from (6) that such vector is horizontal because

$$\frac{d^2X}{ds^2} = 2\sqrt{2} \ ac^3 s(-\sin(cs)^2, \cos(cs)^2, 0).$$

Conversely, if $X(s)$ is a differentiable curve in R^3, s is its arc-length and $d^2z/ds^2 = 0$, then there exist constants m and b such that $z=ms+b$. Since s is the arc length, we have that

$$1 = \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2 = \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + m^2$$

then, $|m| \leqq 1$ and dx/ds, dy/ds satisfy the circunference's equation

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1-m^2 = r^2 .$$

We can then conclude that there exists a function $\phi(s)$ such that

$$\frac{dx}{ds} = r \cos \phi(s)$$

$$\frac{dy}{ds} = r \sin \phi(s)$$

and therefore

$$x(s) = r \int_0^s \cos \phi(\tilde{s})\, d\tilde{s}$$

$$y(s) = r \int_0^s \sin \phi(\tilde{s})\, d\tilde{s} \qquad (7)$$

Comparing (6) with (7) we can see that the projection (x(s),y(s)) is what we could call a generalized cornu's spiral.

3. TRANSVERSE ACCELERATION

Accelerations acting on a wagon at a point of the curve are represented in Fig. 2 (assuming the speed constant and equal to v).

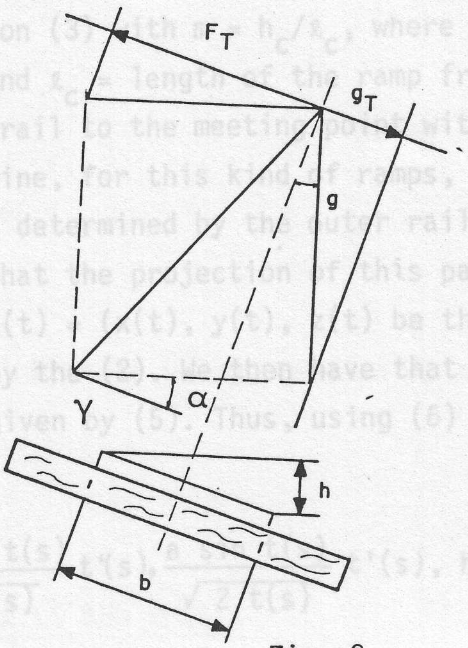


Fig. 2

α = angle due to the superelevation
h = superelevation (mm)
b = gauge

ρ = radius of curvature at the studied point (m)
F = acceleration (m/s²)
g = gravity
V = train's speed (km/h)
γ = transverse acceleration (m/s²)

Then

$$\gamma = F_T - g_T = F \cos\alpha - g \sin\alpha = (F - g \tan\alpha) \cos\alpha$$

Since α is small, we can approximate cos α by 1, thus

$$\gamma = F - g \tan\alpha = \frac{V^2}{13\rho} - 9.81 \frac{h}{b} \qquad (8)$$

4. CURVATURE AT EACH POINT OF THE CURVE

a) Along the circular section of radius R, it is obvious that the curvature κ is constant and equal to 1/R.

b) We shall analise only straight ramps in this work, that is, those satisfying condition (3) with $m = h_c/\ell_c$, where h_c = superelevation of the circular section and ℓ_c = length of the ramp from the point of tangence with the straight rail to the meeting point with the circular section, we shall first determine, for this kind of ramps, the equation X(t)=(x(t),y(t), z(t)) of the curve determined by the outer rail of the ramp. We saw in the previous section that the projection of this part of the rail is a cornu's spiral. Let then X(t) = (x(t), y(t), z(t) be the ramp's equation, with x(t), y(t) given by the (2). We then have that the arc-length s(t) is, for each value of t, given by (5). Thus, using (6) we obtain.

$$X'(s) = \left(\frac{a \cos t(s)}{\sqrt{2\,t(s)}}\, t'(s), \frac{a \sin t(s)}{\sqrt{2\,t(s)}}\, t'(s), h_c/\ell_c\right),$$

but, from (5) we have that

$$t(s) = \left(\frac{1-m^2}{2a^2}\right) s^2 = \frac{\ell_c^2 - h_c^2}{2a^2 \ell_c^2}\; s^2,$$

hence,

$$X'(s) = \left(\frac{(\ell^2_c - h^2_c)^{1/2}}{\ell_c} \cos t(s), \frac{(\ell^2_c - h^2_c)^{1/2}}{\ell_c} \sin t(s), \frac{h_c}{\ell_c} \right),$$

and therefore

$$X''(s) = \frac{(\ell^2_c - h^2_c)^{3/2}}{a^2 \ell^3_c} s \,(-\sin t(s), \cos t(s), 0).$$

Thus, the curvature k(s) at each point of the ramp is

$$\kappa(s) = |X''(s)| = \frac{(\ell^2_c - h^2_c)}{a^2 \ \ell^3_c} s. \tag{8}$$

At the meeting pont of this curve with the circular section we must have

$$\kappa(\ell_c) = \frac{(\ell^2_c - h^2_c)}{a^2 \ \ell^2_c} = \frac{1}{R},$$

from where

$$a^2 = \frac{R(\ell^2_c - h^2_c)^{3/2}}{\ell^2_c}. \tag{9}$$

Substituting a² in (8) by this value, we finally obtain

$$\kappa(s) = \frac{1}{\ell_c R} s \tag{10}$$

5. WEAR-DOWN OF THE RAILS AND WHEELS

According to Novaes [3], practical experience shows that the wear-down of the wheels' brims and rails along curves is a function of:

I - transverse acceleration at the curve,

II - average gross-weight (by axis),

III - number of train passages by unit time.

Using the formula for the wear-down of the circular section given in Novaes [3], and the term corresponding to the ramp, we have that the wear-down function for the whole curve is

$$D(h_c) = 2 \int_0^{\ell_c} K \sum_{i=1}^{M} WT_i . NT_i |\gamma_i(s)| ds + K\, L \sum_{i=1}^{M} WT_i . NT_i |\gamma_i| =$$

$$= 2\, K \sum_{i=1}^{M} WT_i . NT_i \int_0^{\ell_c} |\gamma_i(s)| ds + K\, L \sum_{i=1}^{M} WT_i\, NT_i |\gamma_i|$$

where:

WT_i = i-type train's gross-weight.

NT_i = total number of i-type trains using the given tracks in a year.

$\gamma_i(s)$ = i-type train's transverse acceleration, on the ramp, at the point where the arc length is s.

γ = i-type train's transverse acceleration at the circular section.

K = proportionality constant.

L = length of the circular section.

M = number of trains being considered.

Since the superelevation of the ramp is $h = z(s) = h_c s/\ell_c$ and in the circular section it is h_c we obtain from (8)

$$\gamma_i(s) = \frac{V_i^2}{13} \kappa(s) - 9.81 \frac{h_c s}{\ell_c b} = \left(\frac{V_i}{13 \ell_c R} - 9.81 \frac{h_c}{\ell_c b} \right) s$$

and

$$\gamma_i = \frac{V_i}{13R} - 9.81\,\frac{h_c}{b}$$

Hence,

$$D(h_c) = 2K \sum_{i=1}^{M} WT_i . NT_i \left| \frac{V_i^2}{13\ell_c R} - 9.81\,\frac{h_c}{\ell_c b} \right| \frac{\ell_c^2}{2} + KL \sum_{i=1}^{M} WT_i . NT_i \left| \frac{V_i^2}{13R} - 9.81\,\frac{h_c}{b} \right| = K(\ell_c + L) \sum_{i=1}^{M} WT_i\, NT_i \left| \frac{V_i}{13R} - 9.81\,\frac{h_c}{b} \right| \qquad (11)$$

According to Schramm [4], the following restrictions must be obeyed for dinamical stability and confort conditions:

$-0.98\ (m/s^2) < \gamma_i(s),\ \gamma_i < 0.65\ (m/s^2)$.

Also according to Schramm [4], ℓ_c is a function of the train's speed and of the superelevation. For each kind of train, we have

$$\ell_c = \ell_{c_i} = \begin{cases} 0.01\ V_i\ h_c & \text{if } V_i \geq 40\ km/h \\ 0.4\ h_c & \text{if } V_i \leq 40\ km/h \end{cases} \qquad (12)$$

Since we must use only one value of ℓ_c for all kind of trains considered in Equation (11), we shall choose for each given h_c the largest ℓ_c among the M values given by (12).

6. CONCLUSIONS

Considering the wear-down formula (11), we obtain the optimal value for the superelevation, which is defined as the one value which minimizes $D(h_c)$, obeyed the aforementioned restrictions on $\gamma_i(s)$ and γ_i.

The examples in Section 8 show how such values can be determined using the enclosed computer code.

It can be observed that the method we used permits to obtain conclusions about the interdependence between the superelevation and the other railway parameters.

7. FINAL REMARKS

1 - If instead of considering the cornu's spiral as the transition curve (given by Equation (2)), we take a generic curve (x(t), y(t)) and the corresponding straight ramp, the formula for the transverse acceleration, analog to (9), is

$$\gamma(s) = \frac{V^2}{13} r \mid \phi'(s)\mid - 9{,}81 \frac{h}{b}$$

and it is then possible to extend the results of the present paper.

2 - Taking into consideration the development of microcomputers we believe that implementation of computer programs, like the one enclosed in this paper, as tools for field work can be of great help to engineers.

3 - Formula (11) is similar to the one determined by Novaes [3] for the case of the circular section (ℓ_c=0 m (11)), however, in the present article, ℓ_c depends on h_c in a non-linear fashion.

8. EXAMPLES

otimizacao por busca direta - metodo de fibonacci

sobrelevacao para minimo desgaste

raio da curva 600 angulo 45
comprimento do trecho circular 471.24 m
sobrelevacao recomendada 75.2779 mm
comprimento da rampa 70.0937 m
velocidade maxima 93.1132 km/h

trem - tipo : minerio (carregado)

quantidade 45
peso bruto 12000
vel. maxima de projeto 45
vel. maxima na curva 45

trem - tipo : minerio (vazio)

quantidade 45
peso bruto 2500
vel. maxima de projeto 60
vel. maxima na curva 60

trem - tipo : carga diversa

quantidade 130
peso bruto 6000
vel. maxima de projeto 60
vel. maxima na curva 60

trem - tipo : carga diversa

quantidade 140
peso bruto 3000
vel. maxima de projeto 70
vel. maxima na curva 70

trem - tipo : passageiro

quantidade 90
peso bruto 1000
vel. maxima de projeto 110
vel. maxima na curva 93.1132

trem - tipo : passageiro

quantidade 90
peso bruto 1200
vel. maxima de projeto 110
vel. maxima na curva 93.1132

trem - tipo : passag. rapido

quantidade 60
peso bruto 500
vel. maxima de projeto 140
vel. maxima na curva 93.1132

trem - tipo : passag. rapido

quantidade 60
peso bruto 600
vel. maxima de projeto 140
vel. maxima na curva 93.1132

trem - tipo : carvao (carregado)

quantidade 110
peso bruto 8000
vel. maxima de projeto 60
vel. maxima na curva 60

trem - tipo : carvao (vazio)

quantidade 110
peso bruto 1700
vel. maxima de projeto 70
vel. maxima na curva 70

9. REFERENCES

[1] PACHECO DE CARVALHO,M. Curso de Estradas, estudos, projetos e locação de ferrovias e rodovias. Vol. 1., Ed. Científica, Rio de Janeiro, 1957.

[2] BEZERRA COÊLHO, A. et alli. Desgaste de trilhos. Instituto Militar de Engenharia, Rio de Janeiro, 1982.

[3] NOVAES, A.G. Métodos de Otimização, aplicações aos transportes. Ed. Edgard Blücher Ltda., São Paulo, 1978.

[4] SCHRAMM, G. A geometria da via permanente. Ed. Meridional Emma, Porto Alegre, 1974.

```
10 REM versao final rodando em 25/06/86
20 DIM WT(30),NT(30),V(30),VV(30),R(50),Z$(30),LC(30),LON(10)
30 REM gosub 10000
40 READ TLT$
50 READ GAMA1,GAMA2,SMAX,B,NR
60 INPUT ALFA
70 FOR I=1 TO NR
80 READ R(I)
90 LON(I)=2!*3.1416*R(I)*ALFA/360!
100 NEXT I
110 READ N
120 IF N<=0 THEN 440
130 FOR I=1 TO N
140 READ WT(I),NT(I),V(I),Z$(I)
150 NEXT I
160 FOR K=1 TO NR
170 RR=R(K)
180 LL=LON(K)
190 GOSUB 450
200 X=H
210 GOSUB 640
220 XDUM = 0!
230 FOR I=1 TO N
240 IF XDUM >=VV(I) THEN 260
250 XDUM=VV(I)
260 NEXT I
270 LPRINT TLT$:LPRINT:LPRINT
280 LPRINT "sobrelevacao para minimo desgaste ":LPRINT:LPRINT
290 LPRINT "raio da curva ";RR;"    angulo ";ALFA
300 LPRINT "comprimento do trecho circular ";LL;" m"
310 LPRINT "sobrelevacao recomendada";H;"mm"
320 LPRINT "comprimento da rampa ";MAX;" m"
330 LPRINT "velocidade maxima ";XDUM;"km/h":LPRINT:LPRINT
340 A$="     "
350 FOR I=1 TO N
360 LPRINT "trem - tipo :  "Z$(I):LPRINT:LPRINT
370 LPRINT A$;"quantidade ";NT(I)
380 LPRINT A$;"peso bruto ";WT(I)
390 LPRINT A$;"vel. maxima de projeto ";V(I)
400 LPRINT A$;"vel. maxima na curva ";VV(I):LPRINT:LPRINT
410 NEXT I
420 NEXT K
430 GOTO 110
440 END
450 S1=0!
460 S2=SMAX
470 F=.618
480 FOR I=1 TO 20
490 X1=(1!-F)*(S2-S1)+S1
500 X2=F*(S2-S1)+S1
510 X=X1
520 GOSUB 640
530 OBJ1=DESG
540 X=X2
550 GOSUB 640
560 OBJ2=DESG
```

```
570 IF OBJ1 >OBJ2 THEN 600
580 S2=X2
590 GOTO 610
600 S1=X1
610 NEXT I
620 H=(S1+S2)/2!
630 RETURN
640 DESG=0!
650 FOR L=1 TO N
660 GAMA=(V(L)^2/(13!*RR))-9.81*X/B
670 IF GAMA <=GAMA2 THEN 700
680 GAMA=GAMA2
690 GOTO 720
700 IF GAMA>=GAMA1 THEN 720
710 GAMA=GAMA1
720 GAMA=ABS(GAMA)
730 DESG=DESG+WT(L)*NT(L)*GAMA
740 NEXT L
750 GOSUB 780
760 DESG=DESG*(LL+MAX)
770 RETURN
780 MAX=0!
790 FOR J=1 TO N
800 GA=(V(J)^2/(13!*RR))-9.81*X/B
810 IF GA <=GAMA2 THEN 840
820 GA=GAMA2
830 GOTO 860
840 IF GA >=GAMA1 THEN 860
850 GA=GAMA1
860 VV(J)=SQR((GA+9.81*X/B)*13!*RR)
870 IF VV(J)<=40! THEN 900
880 LR(J)=.01*VV(J)*X
890 GOTO 910
900 LR(J)=.4*X
910 IF LR(J)<=MAX THEN 930
920 MAX=LR(J)
930 NEXT J
940 RETURN
950 DATA "otimizacao por busca direta - metodo de fibonacci"
960 DATA -.98,.65,180.,1600.,4
970 DATA 600.
980 DATA 800.
990 DATA 1000.
1000 DATA 1200.
1010 DATA 10
1020 DATA 12000.,45,45.,"minerio (carregado)"
1030 DATA 2500.,45,60.,"minerio (vazio)"
1040 DATA 6000.,130,60.,"carga diversa"
1050 DATA 3000.,140,70.,"carga diversa"
1060 DATA 1000.,90,110.,"passageiro"
1070 DATA 1200.,90,110.,"passageiro"
1080 DATA 500.,60,140.,"passag. rapido"
1090 DATA 600.,60,140.,"passag. rapido"
1100 DATA 8000.,110,60.,"carvao (carregado)"
1110 DATA 1700,110,70.,"carvao (vazio)"
1120 DATA -1
1130 RETURN
```

Proposta: <Leon>

PNMAC

- financiamento (PIMACC)
- formação de recursos humanos

1. Documento de Trabalho
   (Linhas Mestras)
   - 1.1 Considerações Gerais de MA e CC
   - 1.2. " s/ MAC no Brasil – Histórico
     – Situação atual
   - 1.3. Necessidade de PNMAC (sumário)
   - (*) → 1.4. Proposta de Ação / Cronograma
     - Curto Prazo
     - Longo Prazo

2. Discussão no CNMAC

3. i) Formação de um Painel.
   ii) " de subcomissões
   iii) Discussões regionais.

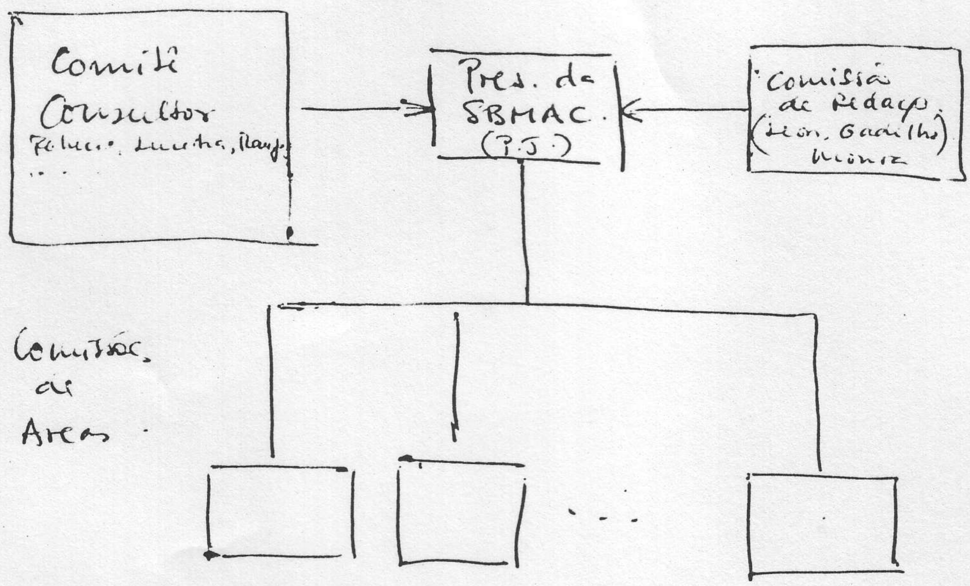
4. Elaboração de uma Proposta de PNMAC pelo Painel.

5. Divulgação da Proposta.

6. Discussão Final no CNMAC'87.

(*)

- Introdução.
- Espectro de Atuação da MA e CC
- Histórico
- Relação com Mat. como Ciência-básica
- " " Ciência da Computação.
- " " Ciências Exatas e da Natureza
- " " Áreas Tecnológicas.
- " " Educação.
- Áreas de Atuação.
- Oportunidades.
- ou Formação profissional
- Política de Financiamento à pesquisa
  - Passado
  - Planejamento Futuro.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