On wavelet techniques in atmospheric sciences .

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Preliminary Motivation

Signals & Representations

- Musical structure — events in time.

We can understand it as a set of musical notes.
They are characterized by

- frequency
- moment it occurs
- duration
- intensity

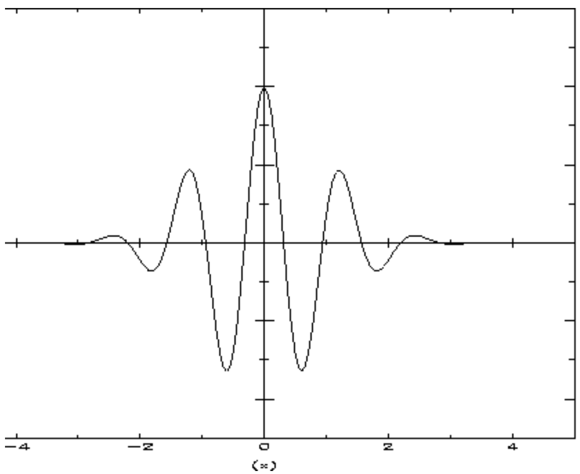
Signals & Representations (cont.)

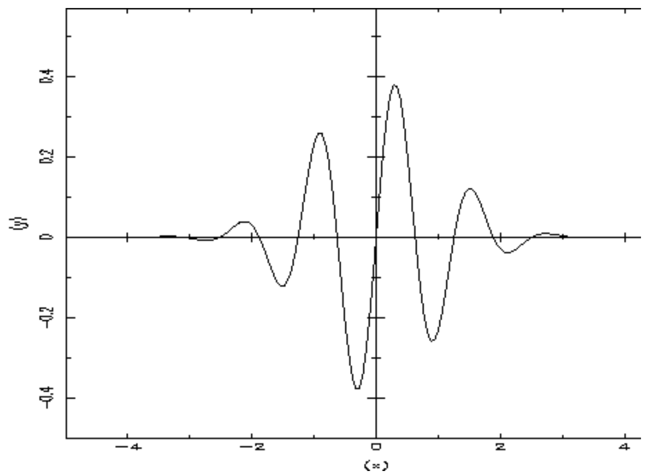
Atmospheric signals & Representations

- Many atmospheric signals also have representations in:
- frequency
- moment it occurs
- duration
- intensity

ondelette, wavelet , ondeleta

- Main idea is “small waves“. What means it?
- Waves that are localized, i.e., wave-like functions whose values increase and decrease in a short period of the domain.





Morlet

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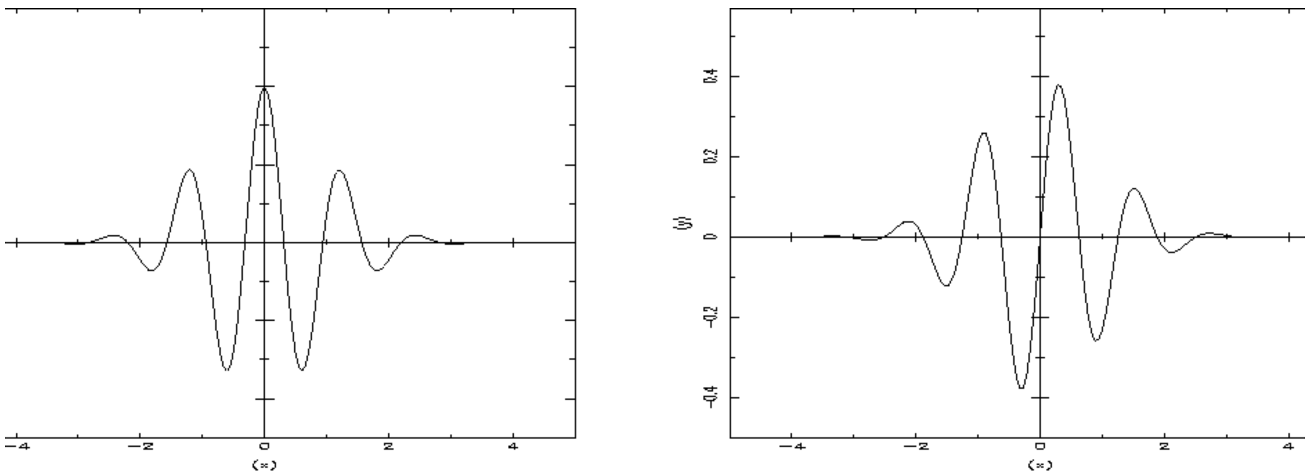


Morlet

ondelette, wavelet , ondeleta

In order to a function be called a wavelet it must satisfy the following conditions

0) The integral of the wavelet function, usually denoted by ψ, must be zero

$$\int_{-\infty}^{\infty} \psi(t)\, dt = 0.$$

This assures that the wavelet function has a waveform shape and it is known as the admissibility condition.

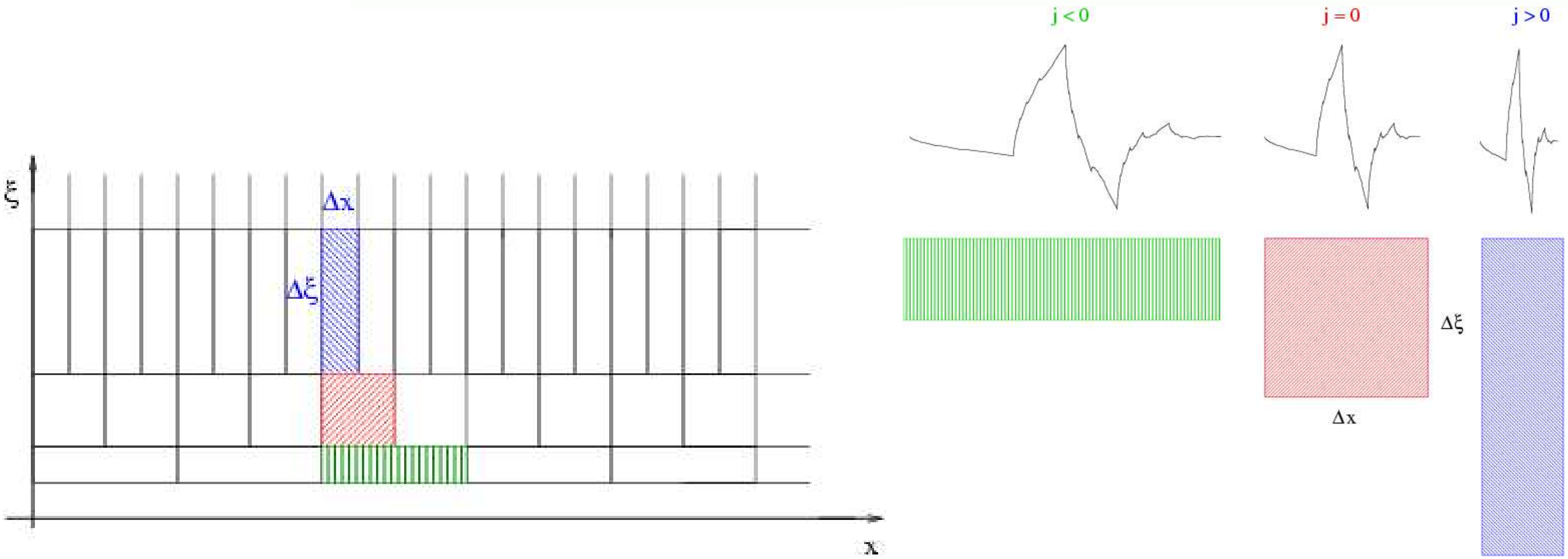
ondelette, wavelet , ondeleta

2) The wavelet function must have unitary energy

$$\int_{-\infty}^{\infty} |\psi(t)|^2 \, dt = 1.$$

This assures that the wavelet function has compact support or has a fast amplitude decay (in a physical vocabulary *e-folding time*), warranting a physical domain localization.

Double localization property



Proportional variations of time/space and frequency/scale intervals.

- mother–wavelet
- dilatation for the mother–wavelet
- contraction of the mother–wavelet

Wavelet Transform

Tool for non–stationary signal analysis

Wavelet Transform

- extract frequency information and where it occurs
- detect localized structures.

Wavelet Transform

CWT

$$\mathfrak{W}_f^{\psi}(a,b) = \int_{-\infty}^{\infty} f(u)\bar{\psi}_{a,b}(u)\,du \qquad a > 0,$$

where

$$\psi_{a,b}(u) = \frac{1}{\sqrt{a}}\,\psi\left(\frac{u-b}{a}\right)$$

- a is a scale parameter
- b is the translation parameter

Dilation & Translation

Parameter a:

- when $a > 1$ – *wavelet* –mother is dilated
- when $a < 1$ – *wavelet* –mother is contract
- it is possible to analyze global an local aspects of the signals around b
- as b shifts, f is locally analyzed nearby b.

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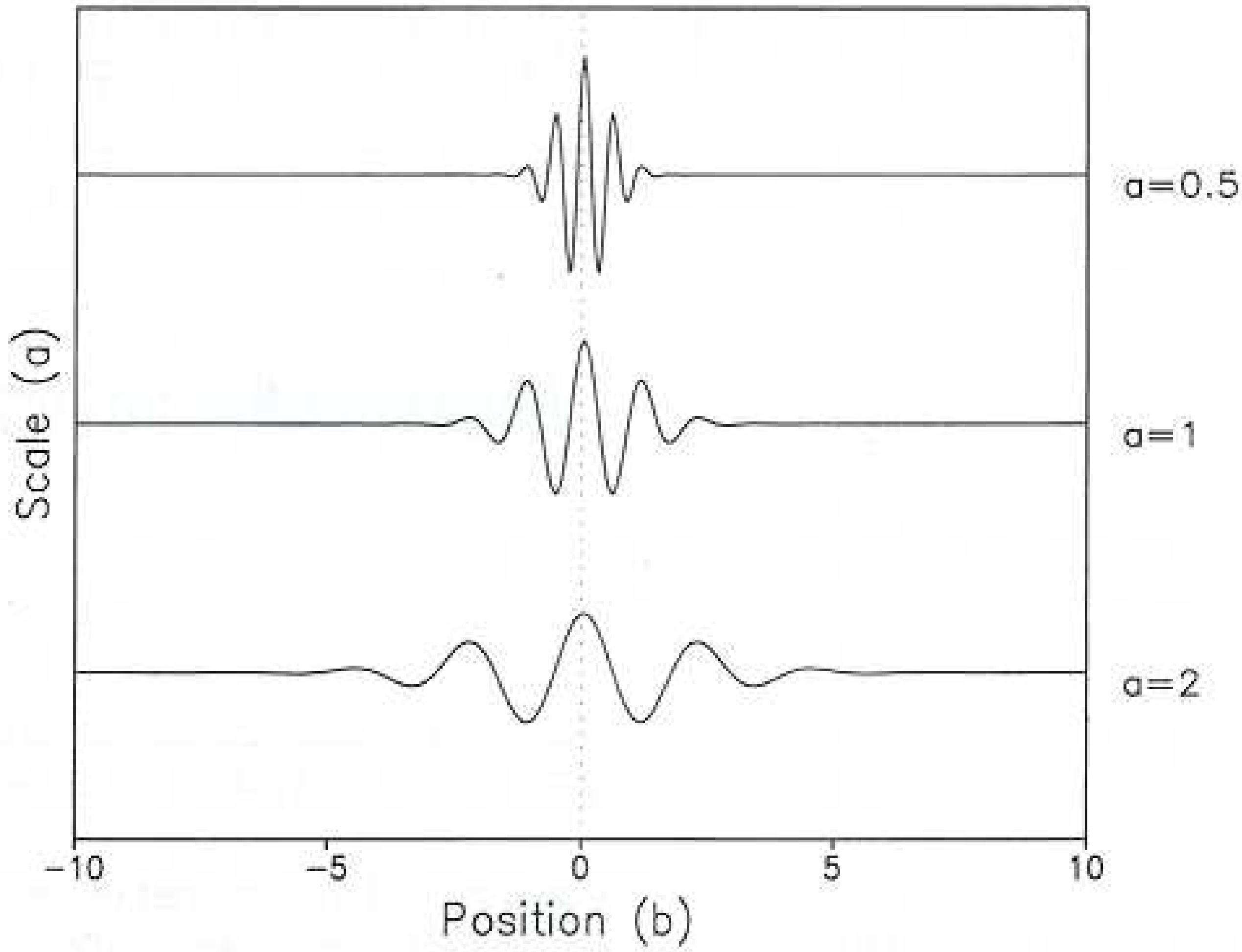
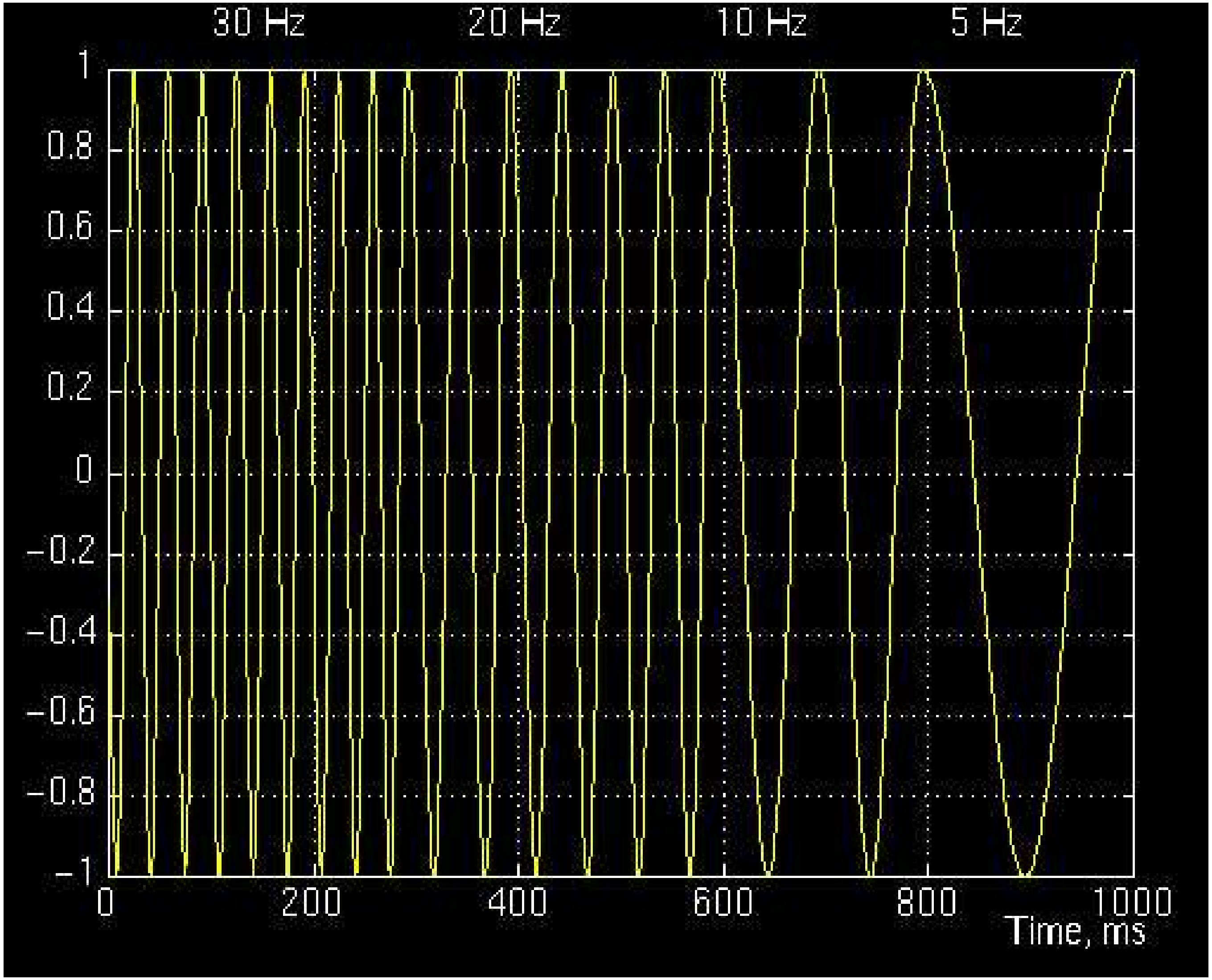


Signal & wavelet transform



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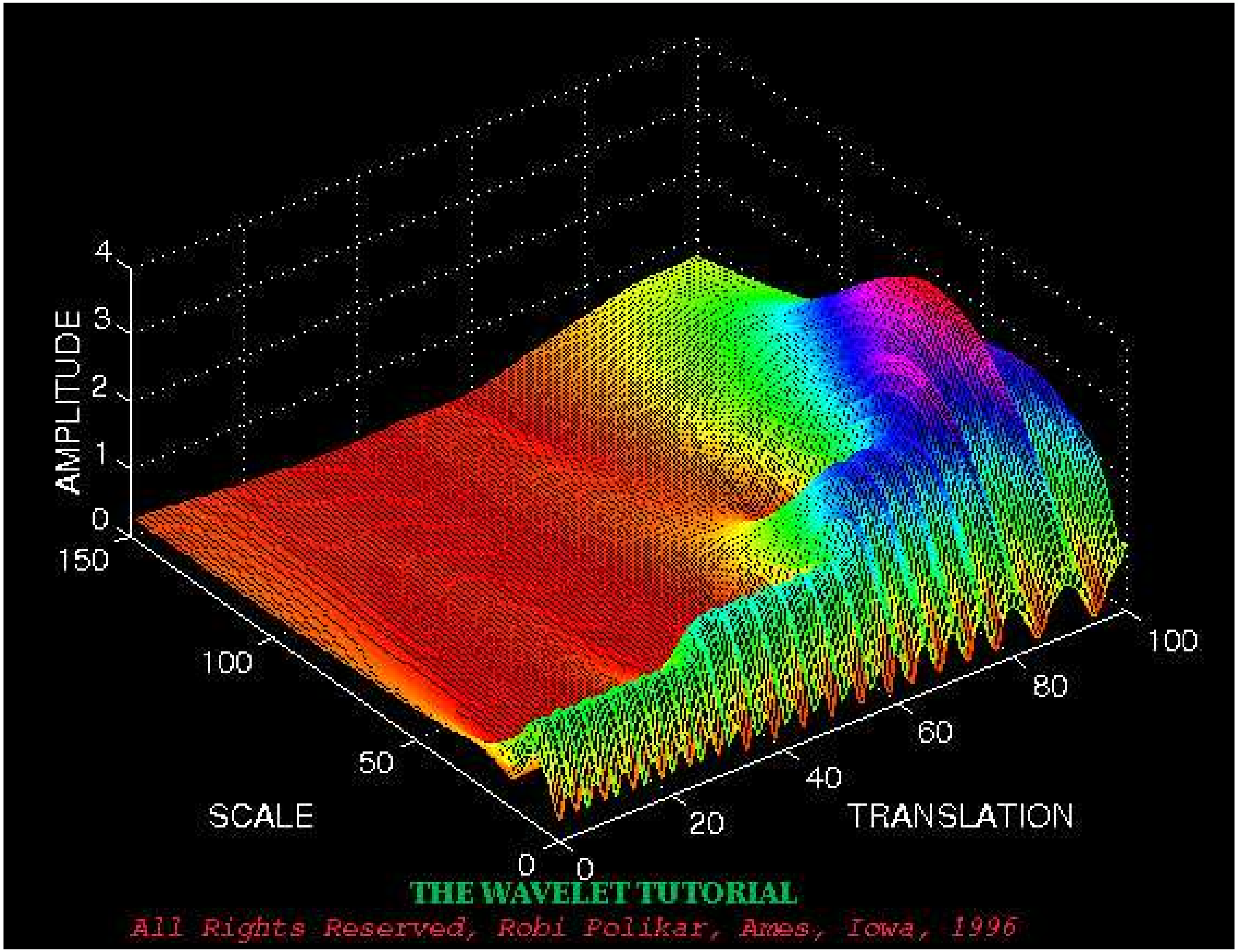


Continuous wavelet transform (CWT *)*

The* CWT *is equivalent to a mathematician microscope, whose magnification is given by the inverse of the dilation parameter and the optical ability is given by the choice of the mother-wavelet function

(Foufoula & Kumar,1994)

Continuous wavelet transform (CWT)

- it is a linear and covariant under translation and dilatation transform.
- the scale and localization parameters assume continuous values
- ICWT

$$\mathfrak{IW}_f^{\psi}(b) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{1}{a^2} \mathfrak{W}_f^{\psi}(a,u)\, \bar{\psi}_{a,b}(u)\, da\, du,$$

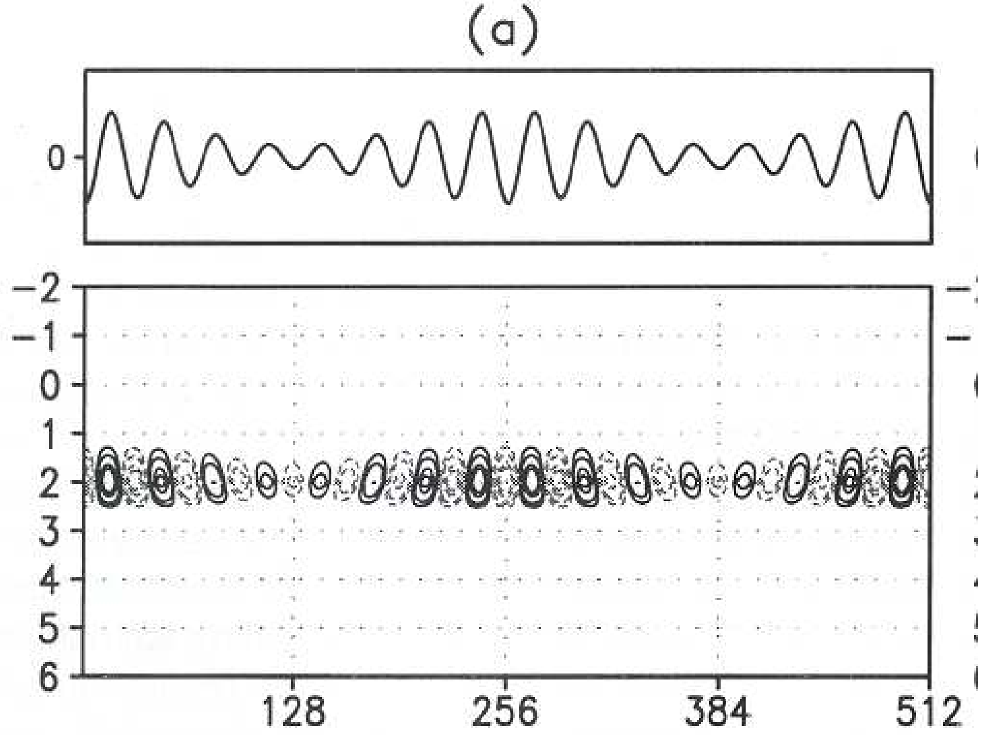
C_ψ is a constant that depends on the chosen wavelet function.

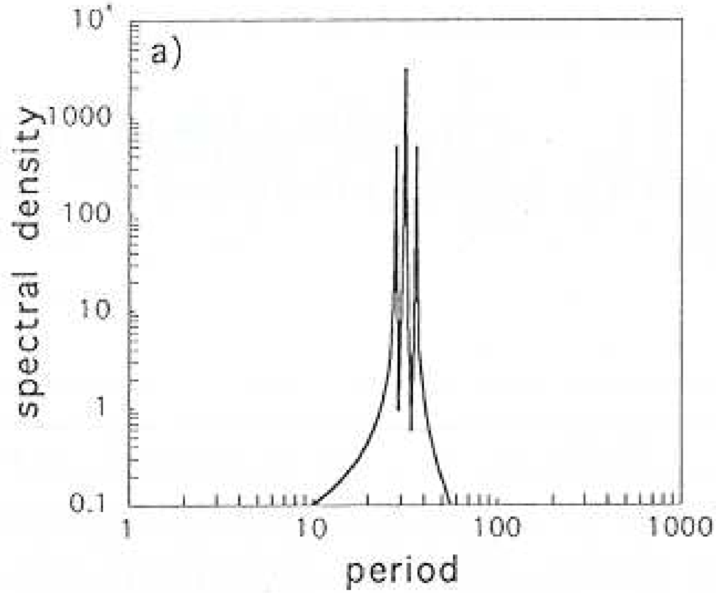
Scalogram

- the wavelet transform is a transform that preserves the energy
- the squared modulus of the wavelet coefficients of the CWT is called scalogram
- the product of two CWT of distinct functions is called cross-scalogram Flandrin(1988)
- the scalogram informs and which scales participate in the processes depicted by the signal
  - if the analyzed signal has multi-scale characteristics
  - which scales participate in the processes depicted by the signal

Synthetic Signal & **CWT**

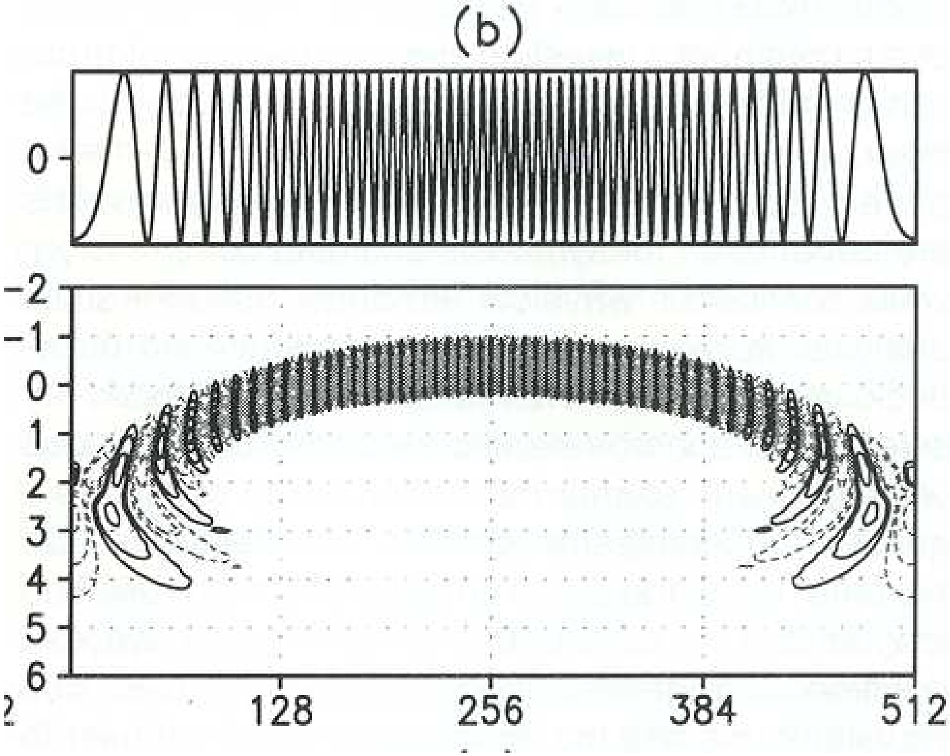
Amplitude modulate signal

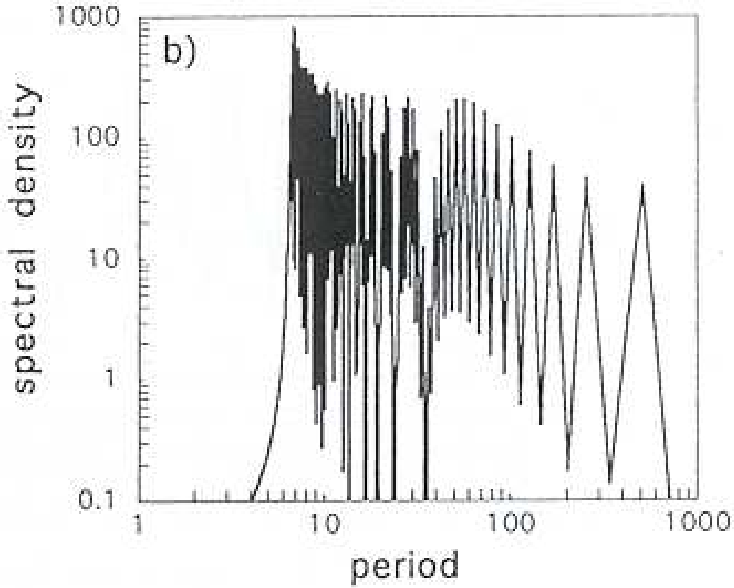




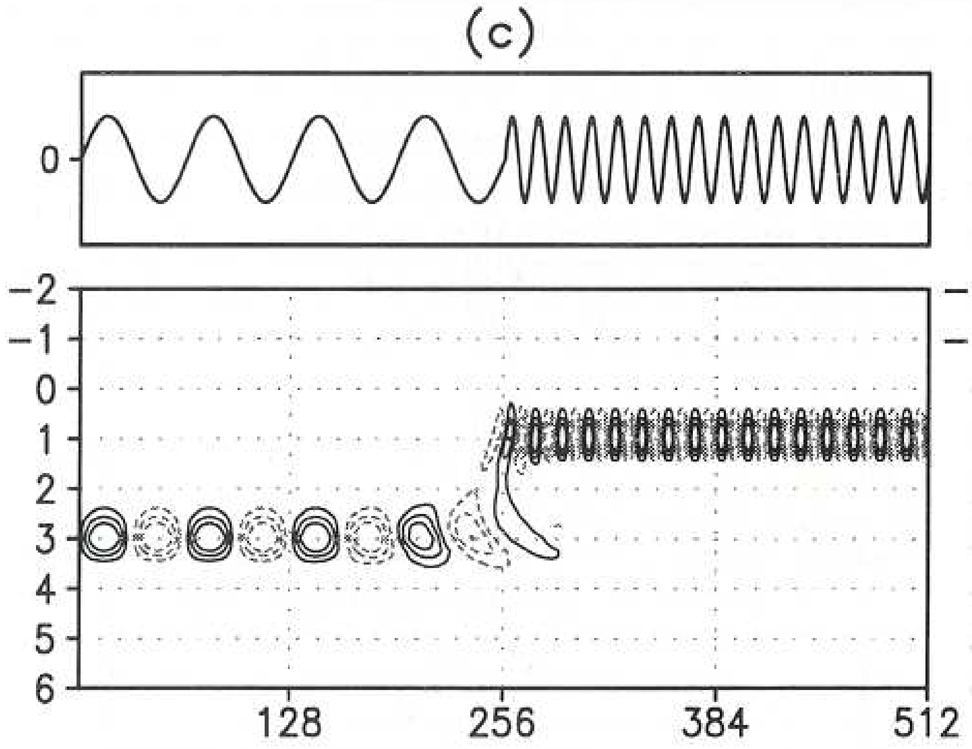
(Weng&Lau,1995)

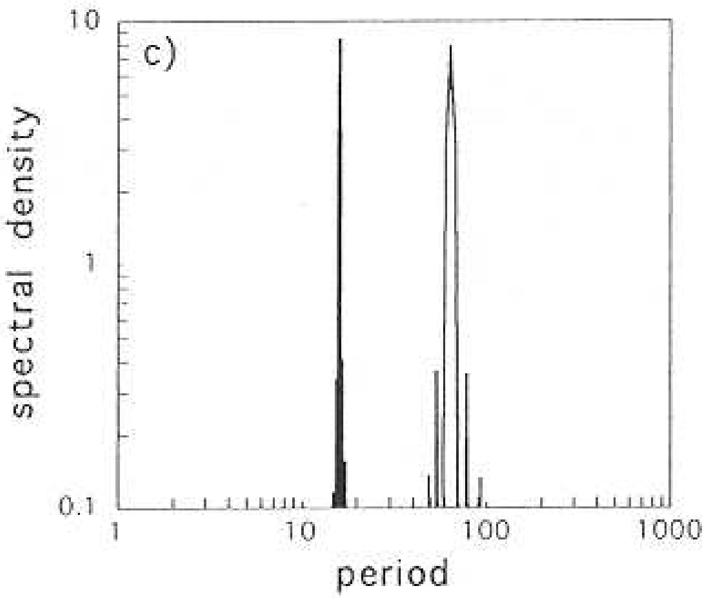
Frequency modulate signal



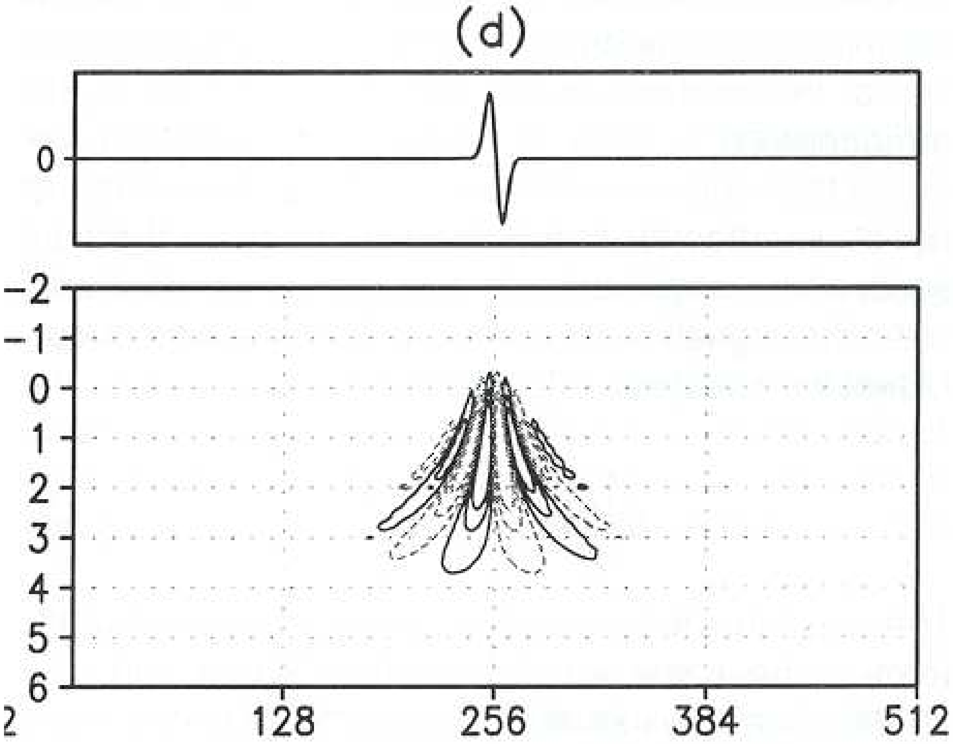


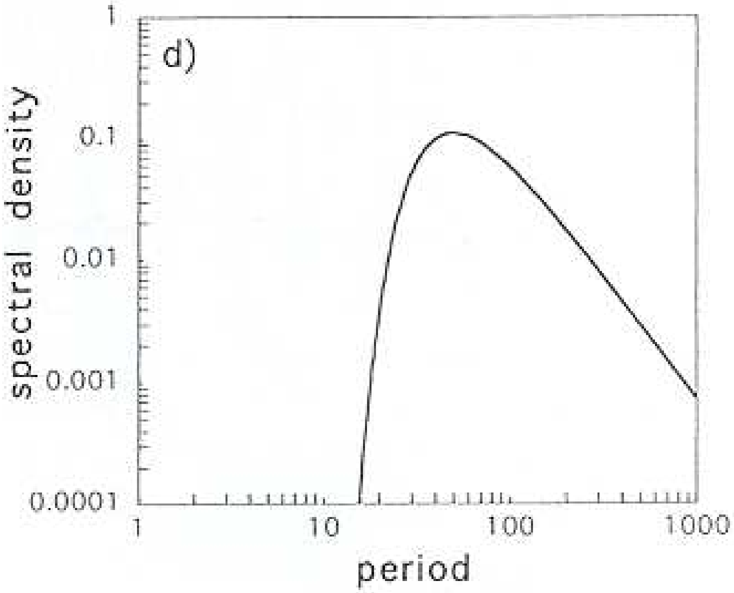
Frequency localized signal





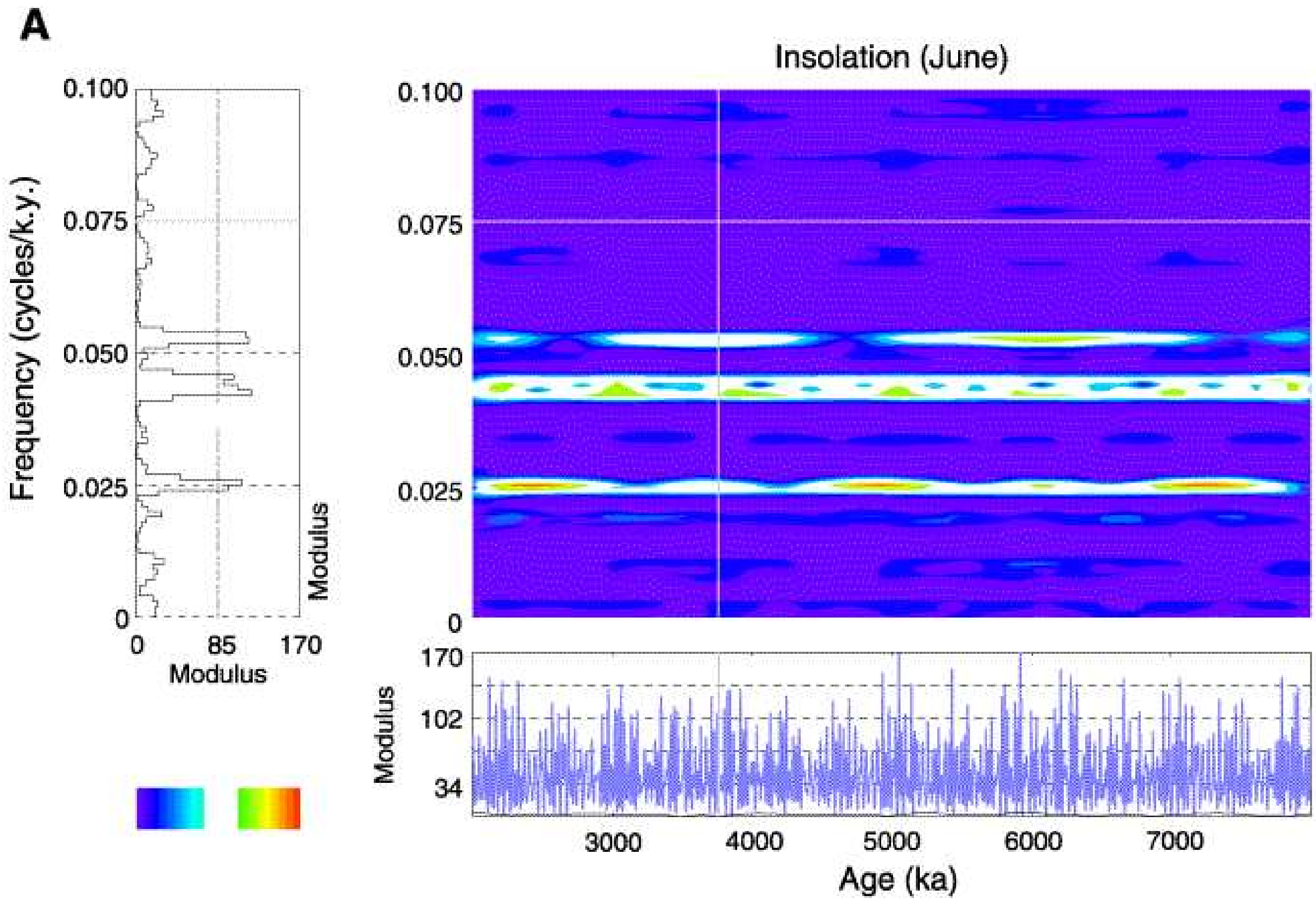
Time localize signal





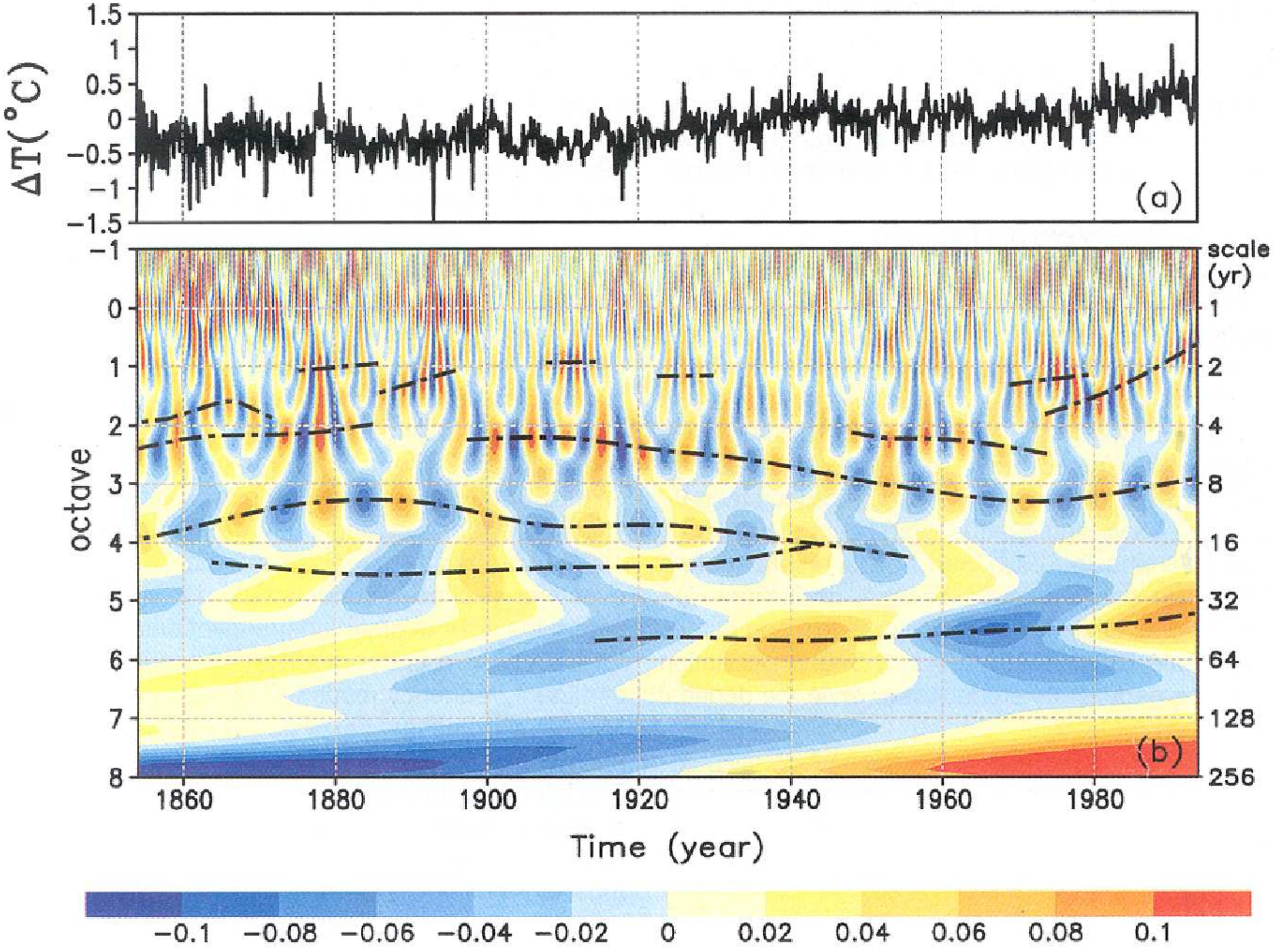
Atmospheric signals & CWT

Ex: Insolation Wavelet signals



SOURCE: `http://www-odp.tamu.edu/publications/178_SR/chap_32/c32`

Temperature signals



Discrete wavelet Transform

DWT

DWT is defined as

$$\mathfrak{D}f_k^j = 2^{-\frac{j}{2}} \int_{-\infty}^{\infty} f(u)\psi_k^j(u)\, du,$$

$$\psi_k^j(u) = 2^{-\frac{j}{2}}\ \psi\left(2^{-j}u - k\right).$$

- uses discrete values of scale (j) and localization (k)
- one may have redundant representations or not
- to avoid redundancies, one can chose *wavelet* functions that form an orthogonal basis

Orthogonal wavelet functions

Such wavelet functions are orthogonal with its respective functions translated and dilated They form a orthogonal system, i.e.,

$$\int_{-\infty}^{\infty} \psi(2^j x - k)\, \psi(2^\ell x - n)\, dx = \begin{cases} 2^{-j} & \text{if } j = \ell \text{ and } k = n, \\ 0 & \text{otherwise.} \end{cases}$$

Signals $f(t)$ are represented by series

$$f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_k^j \, \psi_k^j(t),$$

$\psi_k^j(t) = \psi(2^j t - k)$ are the *wavelet* functions and d_k^j are called the *wavelet* coefficients

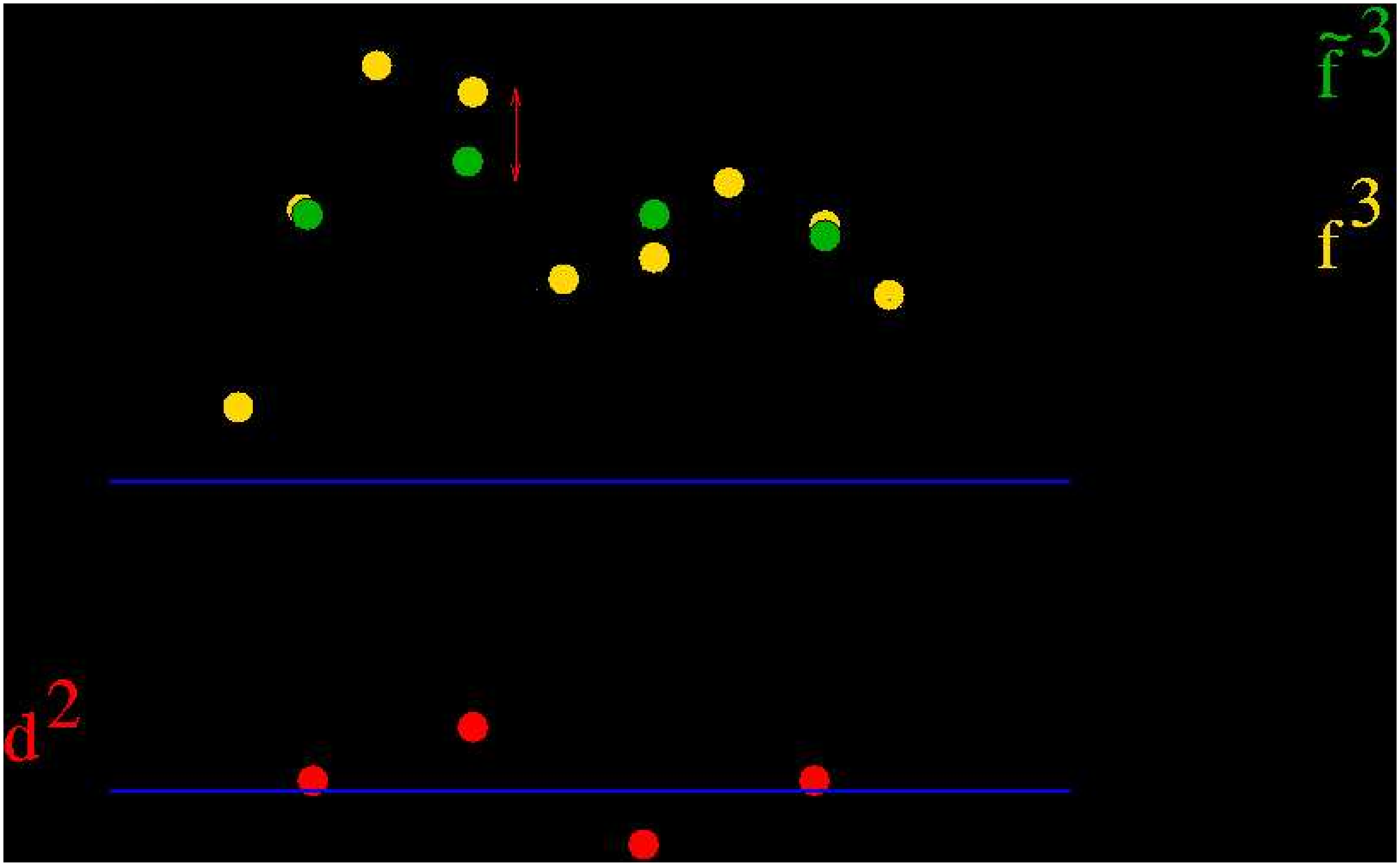
$$d_k^j = \int f(u) \, \psi_k^j(u) du.$$

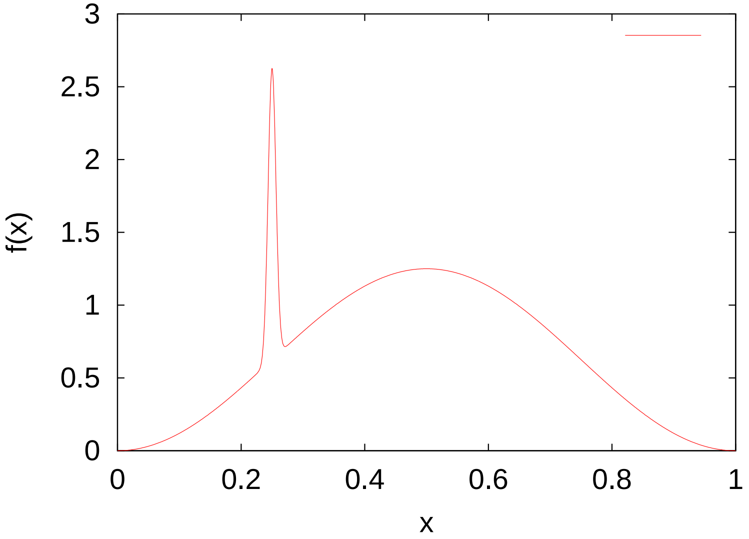
Wavelet coefficients & abrupt signal variations

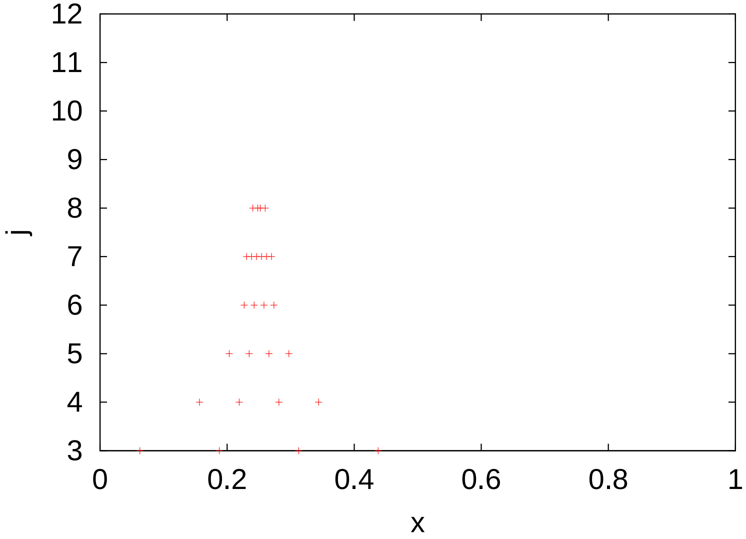
- As a property of the wavelet analysis, it is possible to show that the amplitude of the wavelet coefficients is associated with abrupt signal variations or ”details´´ of higher frequency

(Meyer,1990;Daubechies,1992;Chui:1992b)

wavelet coefficient




f(x)
x
3
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1


j
x
12
11
10
9
8
7
6
5
4
3
0
0.2
0.4
0.6
0.8
1

Some orthogonal wavelet functions

wavelet Haar

$$\psi(t) = \begin{cases} 1, & 0 \leq t < 1/2 \\ -1, & 1/2 \leq t < 1 \\ 0, & \text{otherwise.} \end{cases}$$

- The DWT using Haar wavelet detects signal abrupt variations very well, i.e., , one localization feature in the physical space.

Multiresolution analysis - A tool to design wavelet

- It is possible to build up wavelet functions using a mathematical tool known as multi-resolution analysis formed by a pair $\{V^j, \phi^j\}$, in such a way that there are sequences of embedded approximating spaces $V^j \in V^{j+1}$ and the functions ϕ_k^j formed a Riesz basis for V^j

(Mallat,1989:Daubechies,1992; and many other).

In this technique, a mother-*wavelet* function is generated from a scaling function. It obeys the scale relation

$$\phi(x) = 2\sum_k h(k)\phi(2x-k),$$

$\phi(x)$ is known as the scale function, and $h(k)$ is a low pass filter. Then the mother–wavelet functions are build as

$$\psi(x) = \sum_k g(k)\phi(2x-k),$$

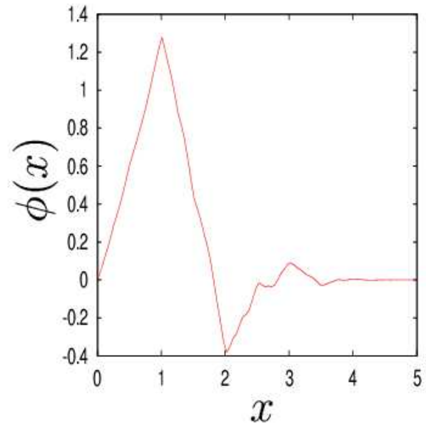
where $g(k) = (-1)^{k+1}h(1-k)$ is a high pass band filter. MRA allows that a basis with compact support and arbitrary smoothness degree can be build up.

Scale coefficient $h(k)$ for Daubechies orthogonal functions

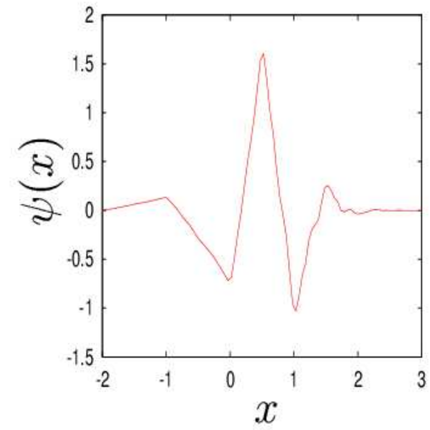
	$K = 2$	$K = 3$	$K = 4$
k	$h(k)$	$h(k)$	$h(k)$
0	0.341506350946110	0.235233603892082	0.162901714025649
1	0.591506350946109	0.570558457915722	0.505472857545914
2	0.158493649053890	0.325182500263116	0.446100069123380
3	-0.0915063509461096	-0.0954672077841637	-0.0197875131178224
4		-0.0604161041551981	-0.132253583684520
5		0.0249073356548795	0.0218081502370886
6			0.0232518005354909
7			-0.00749349466518071

orthogonal scale and wavelet function families.

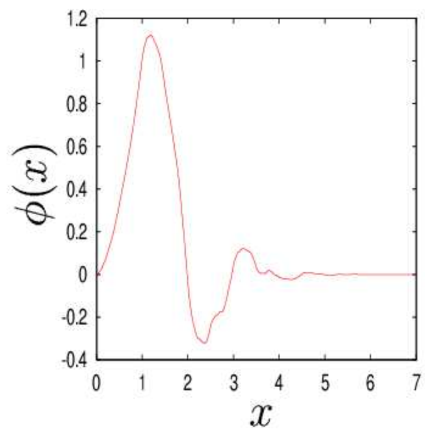
(a) $\phi_3(x)$



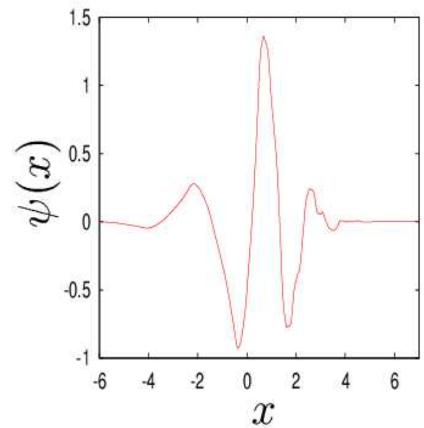
(b) $\psi_3(x)$



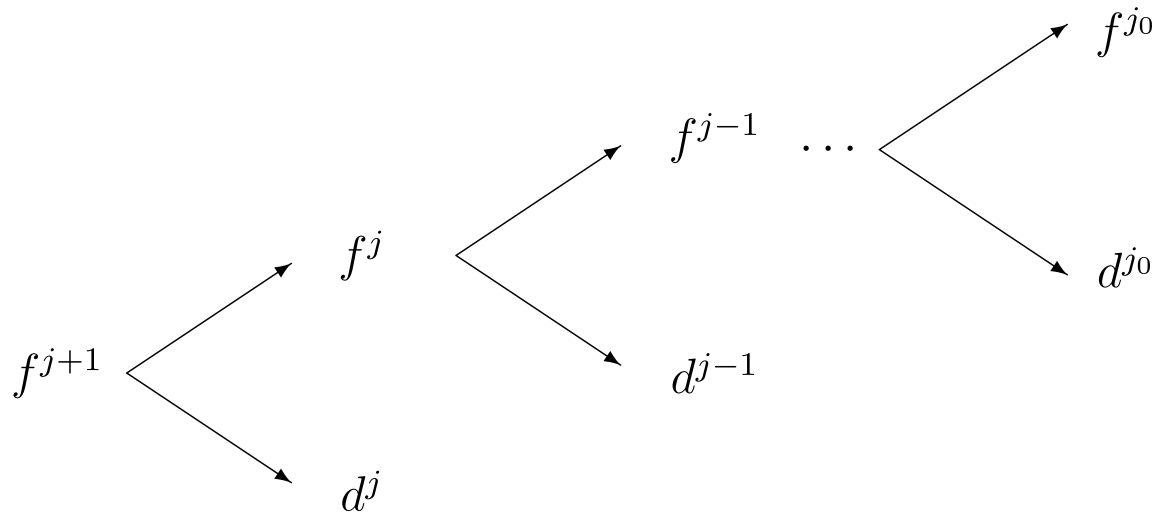
(c) $\phi_4(x)$



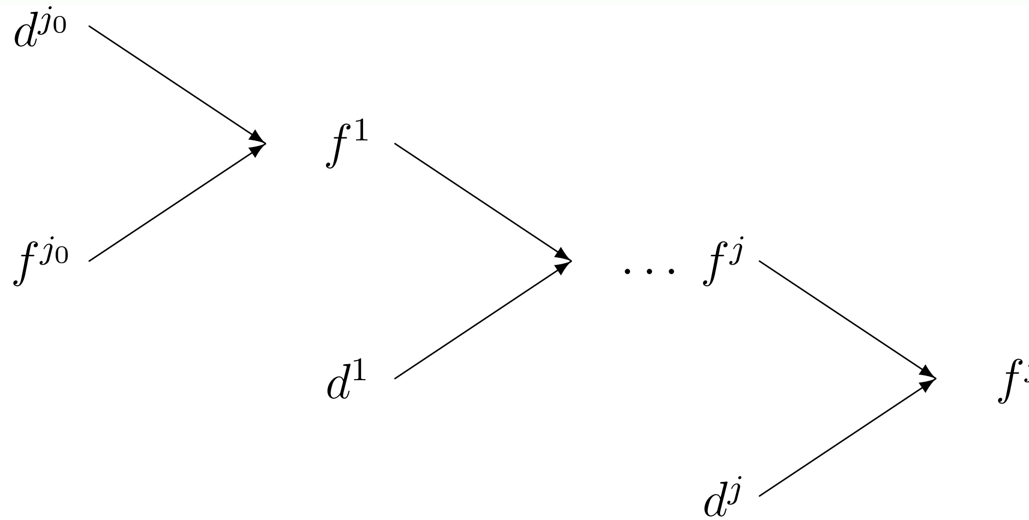
(d) $\psi_4(x)$



DWT



IDWT

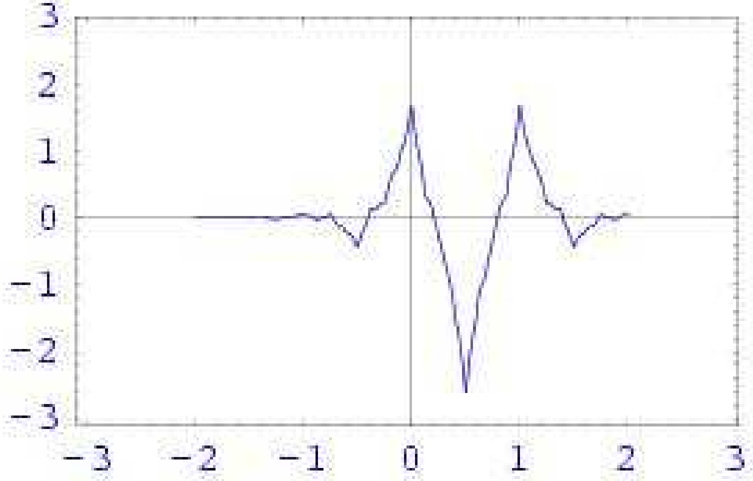


Daubechies biorthogonal wavelet function

- In order to get symmetry, it has been developed a procedure using two multi-resolution analysis with defined bi-orthogonal relations, what has led to the construction of the Daubechies bi-orthogonal families. This kind of wavelet family is usually used in numerical analysis.

(Daubechies:1992)

biorthogonal wavelet function families {2,4}.



Variance analysis

Focusing on the measurement and characterization of the local kinetic energy in each scale in turbulence flow, the variance wavelet analysis or the wavelet spectrum has been originally defined by Meneveau(1991) as

$$S(a) = \int_{-\infty}^{\infty} \mathfrak{W}_f^{\psi}(a,t)dt.$$

- it is also possible to derive the wavelet spectrum starting from the DWT
- total energy contained in each scale j is expressed by

Variance analysis (cont.)

$$S_w^j = \frac{ds}{2\,\pi\,ln(2)}\,2^{-(J-j)}\sum_{k=1}^{2^{J-j}}\left[\,d_k^j\,\right]^2,$$

with the wave number

$$k^j = \frac{2\pi}{2^j\,ds},$$

- considering signals with zero average and $N = 2^J$ elements
- where ds is the interval of the observed samples

Katul et all(1994),Percival & Walden(2000)

Variance analysis (cont.)

- In terms of the scale the resolution performance is only on octaves, in the Fourier analysis, the wavenumber is spaced linearly.
- The deviations of the energy around its mean value can be quantified by the variance of $E(k^j)$, which is essentially a fourth order moment (flatness) of the wavelet coefficient

$$\sigma_E(k^j) = \frac{ds}{2\,\pi\,ln(2)\,2^{(J-j)+1}} \left\{ \sum_{k=1}^{2^{J-j}} \left[\, d_k^j \,\right]^4 - \sum_{k=1}^{2^{J-j}} \left[\, d_k^j \,\right]^2 \right\}^{\frac{1}{2}} .$$

Analogously, it is also possible to define a wavelet co-spectrum of two functions f and g of its wavelet coefficients dj in j scales, by means of

$$\mathcal{C}_w^j = \frac{ds}{2^j \, ln(2)} \sum_{k=1}^{2^j} d_k^{j,(f)} \; d_k^{j,(g)},$$

where $d_k^{j,(f)}$ and $d_k^{j,(g)}$ are the wavelet coefficients obtained in the wavelet transform of the functions f an g respectively.

micro-meteorological tower in Rebio Jaru (Amazon region)

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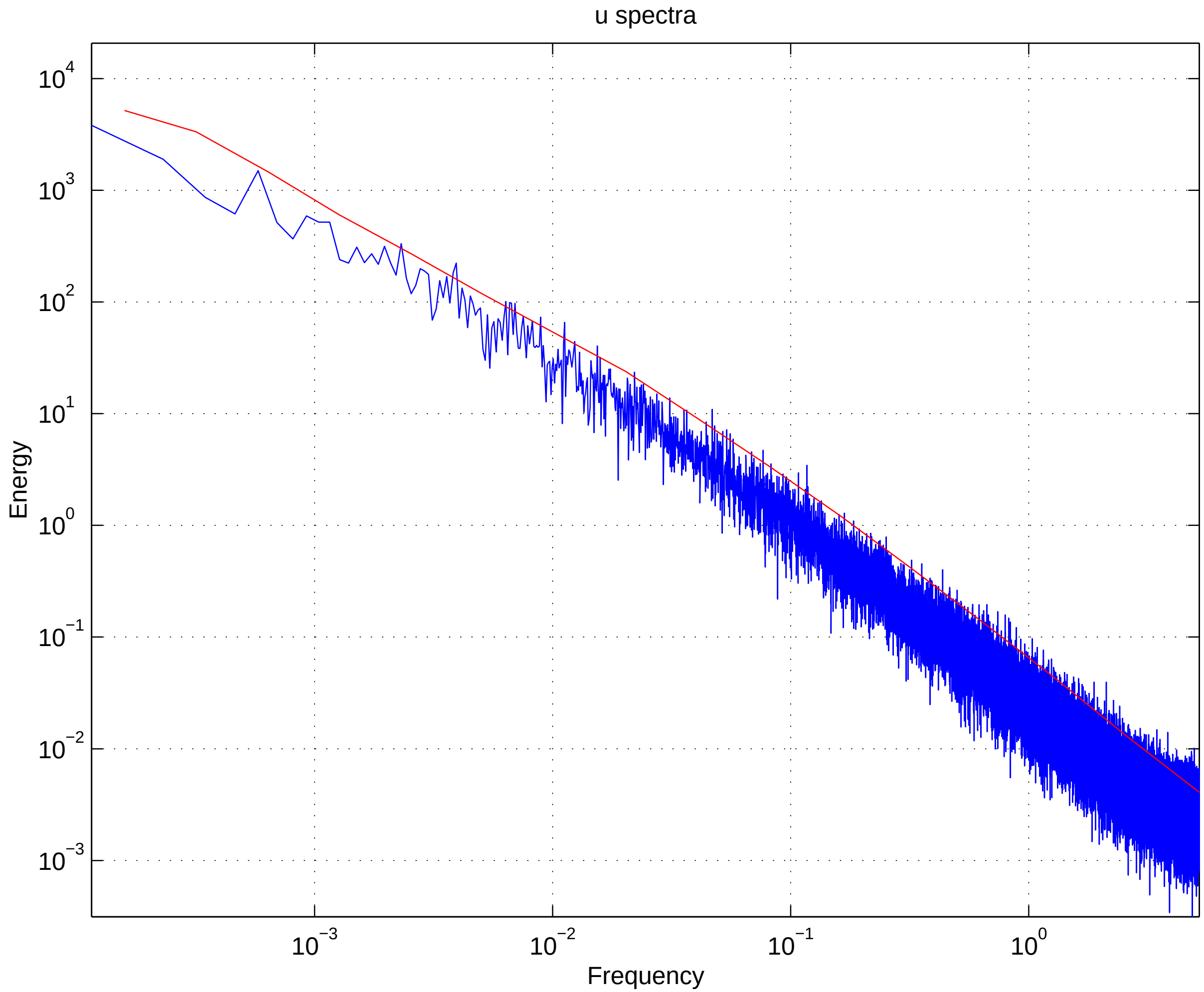
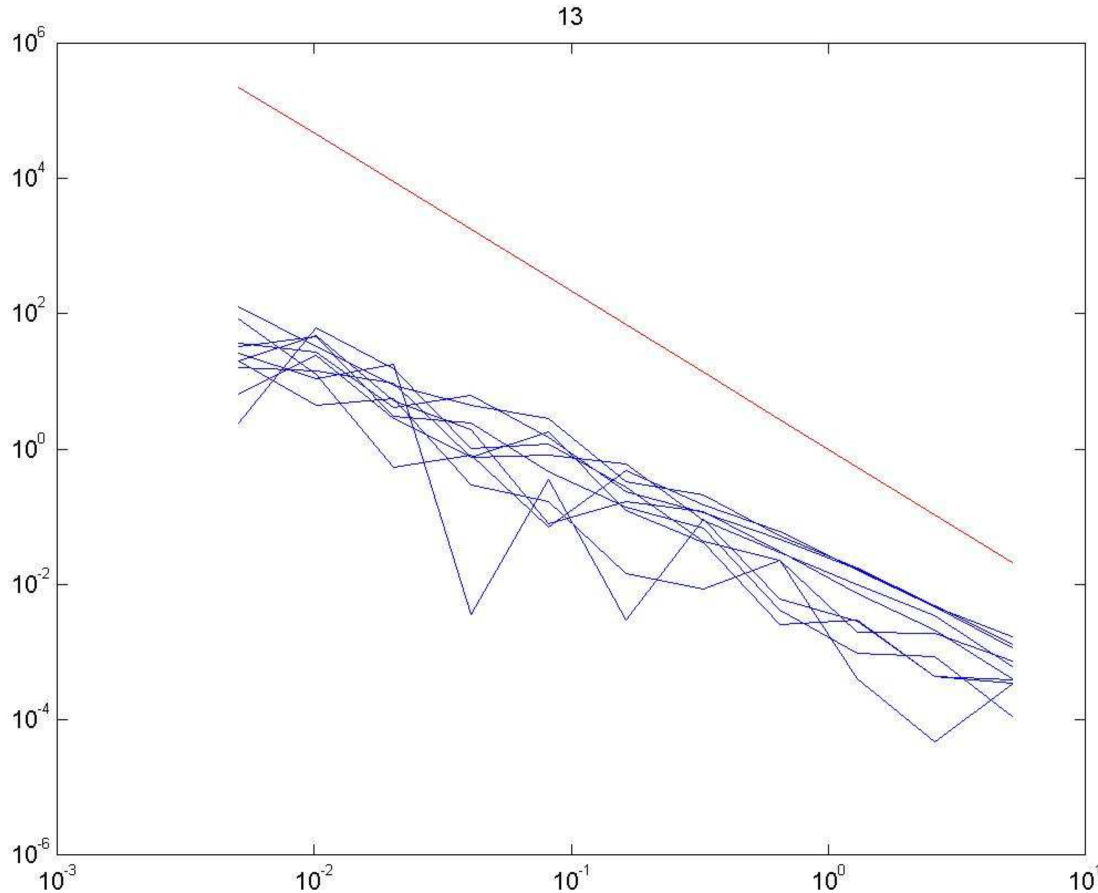
wavelet spectrum of the zonal wind component



micro-meteorological tower in Rebio Jaru (Amazon region)

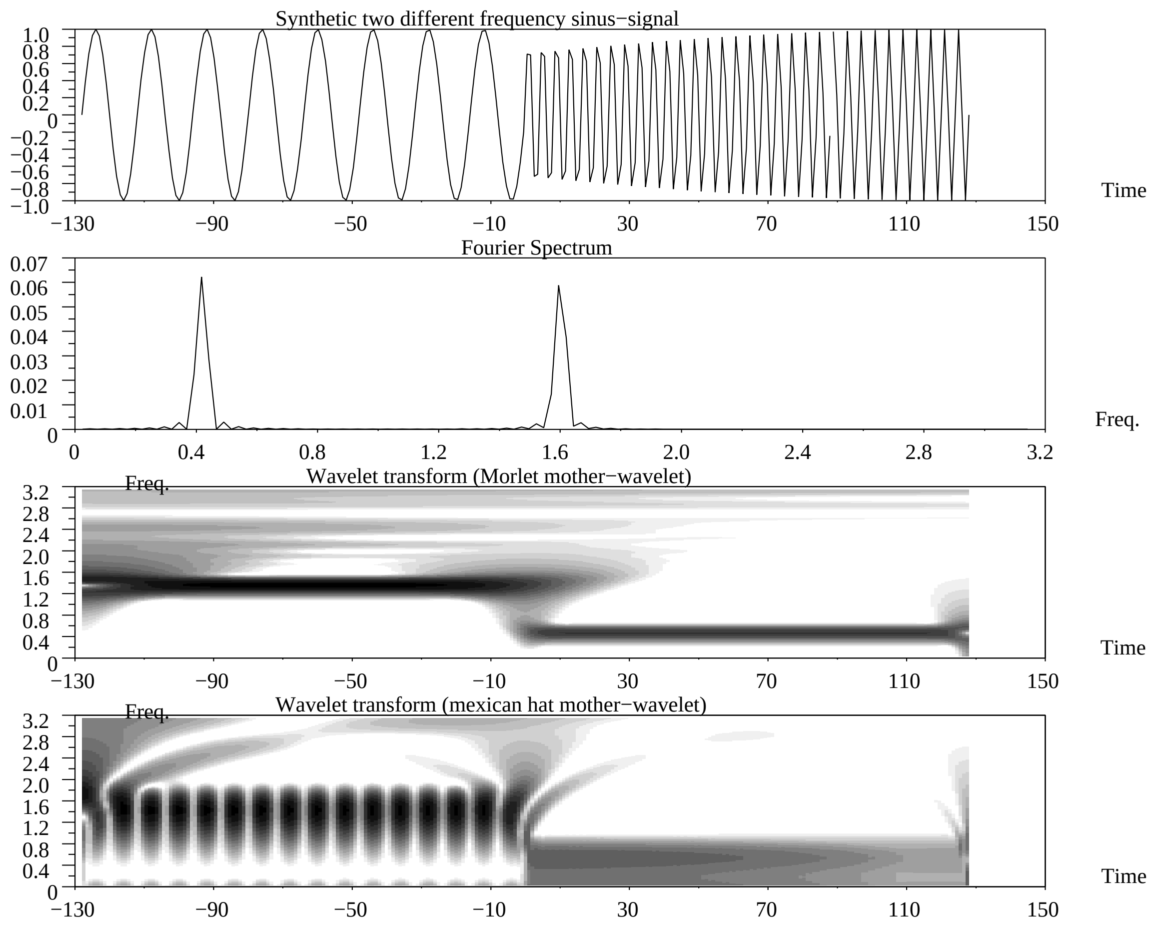
Co-spectrum of temperature and vertical wind component of turbulent signals and a power law



How to chose a wavelet function?

- An issue that is always emerging in the application of wavelet techniques is the choice of the wavelet function appropriate to an specific signal
- There is not a unique answer for It!!

How to chose a wavelet function?



How to chose a wavelet function?

Some recommendations can be useful:

- the shape of the chosen wavelet function must translate the characteristics of the time series
  - to represent a time series with abrupt variations or steps ⇒ Haar *wavelet*
  - in the analysis of time series with smoother variations ⇒ Morlet e mexican hat *wavelet*

How to chose a wavelet function?

- when the analysis is focused in amplitude and phase changes complex *wavelet*
- this helps to retrieve the oscillatory behavior of the data
- in an exploratory analysis of data, non-orthogonal wavelet functions seem helpful, because they allow a redundancy in the information.

How to chose a wavelet function?

- to synthesize data and make compressions, orthogonal wavelet functions are used, since they represent the signal in a more compact way.
- when a quantitative information about a process is needed, orthogonal wavelet functions are the better choice (Kumar & Foufoula,1997).

How to chose a wavelet function?

- Katul & Vidakovic,(1996,1998) e Vidakovic(2000)
- What Daubechies' family should we choose?

How to chose a wavelet function?

- When only the wavelet spectrum is analyzed qualitatively, this choice does not seem to effect the results. This has been established by Katul and Torrence & Compo, for turbulence data and series of climatic data.

Atmospheric Applications

Essential usage

- as an integration nucleus of the analysis to get information about the processes
- as a representation or characterization basis of the processes

Some select papers

Select papers

- Some selected papers, here shortly described, reveals applications in a wide range of phenomena.
- Ranging from issues related to atmospheric-ocean interactions to nearby space conditions, all aspects are related to the atmosphere as a whole.

Select papers

- WengLau(1994) — organization of the tropical convection in the west Pacific,
  - create synthetic time series of dynamical systems, with double period
  - subsequently they applied this recognition pattern to an infrared data series, obtained from satellite high resolution images
  - using both Haar wavelet and Morlet wavelet

Select papers

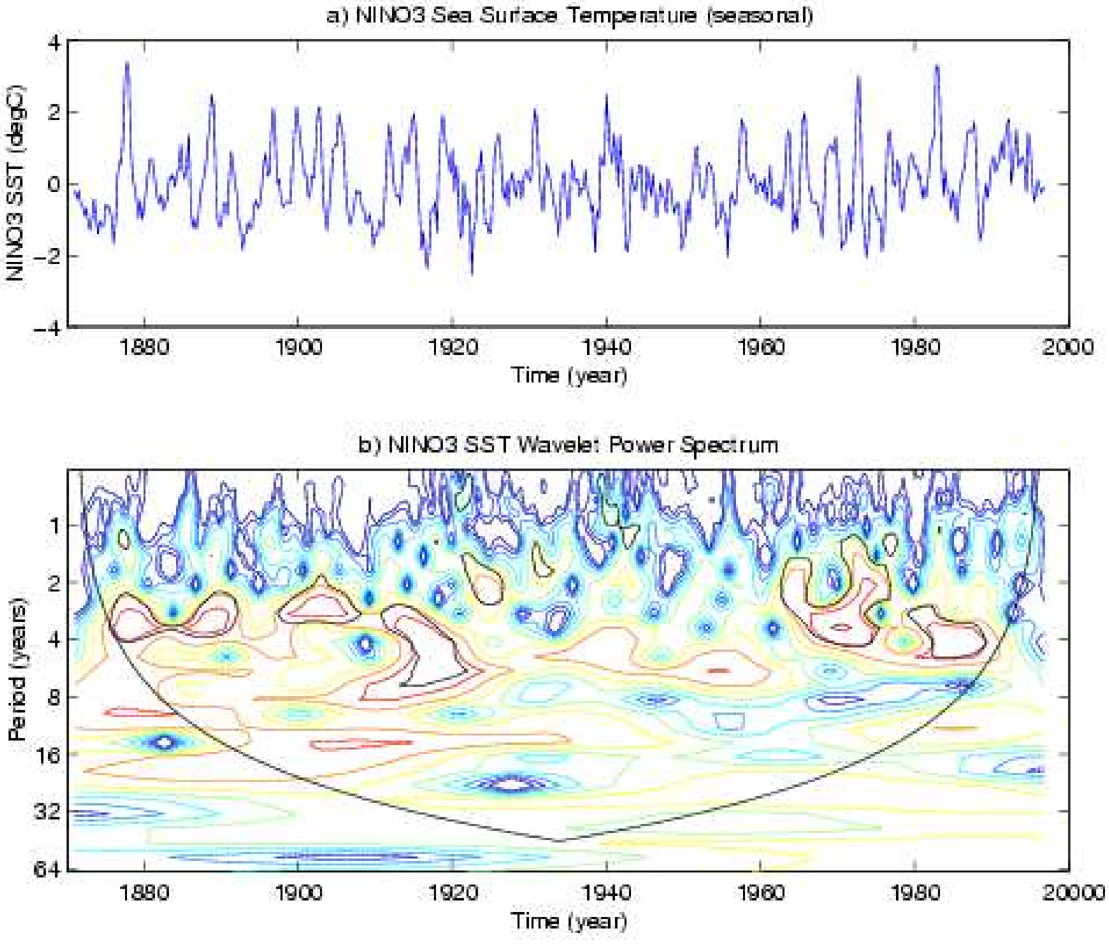
- Briggs & Levine(1997) have applied DWT techniques in an exploratory analysis, in checking forecasting fields since the conventional measurements still reveals inadequate
- this technique allowed a convenient compacting and filtering the fields partitioning, what helps in the physical interpretation of the results

Select papers

- Torrence & Compo(1998) - have investigated time series associated to this phenomena to assess and compare wavelet analysis techniques with the results already known
- they have implemented a Monte Carlo technique to set up the confidence limits in the variance wavelet analysis

Select papers

Periods between 2-8 years are founding in the data sets before 1920 and 2-4 years after 1960



Select papers

- the DWT can also be used to discriminate mesocyclones in Doppler radar data Desrochers & Yee (1999)
- the DWT can also be used to characterize the structures of convective systems Yano et all(2001a,b)
- In relation to the solar irradiance and climatic reconstructions, Oh et all(2003) have conducted a multi-resolution time series analysis.
- The decomposition through the DWT has been made in order to facilitate the identification of the common characteristics between these time series and the climatic forcing physically associated.

Select papers

The CWT together with the Morlet wavelet has been used in the detection and processing of magnetotelluric transients originated by atmospheric electrical discharges Zhang(1997).

- examine the transient signals in audio-frequencies, the dominant energy sources are concentrated in thunderstorms both nearby and at great distances
- amplitude and phase analysis of such pulses, distinguishing them from the noisy background

Select papers

Ageyev(2003)

- to analyze sferics signals produced by lightnings
- in order to obtain information on the electromagnetic field, morphological structure of the ionized channel and on the behavior of the discharge electric current

Select papers

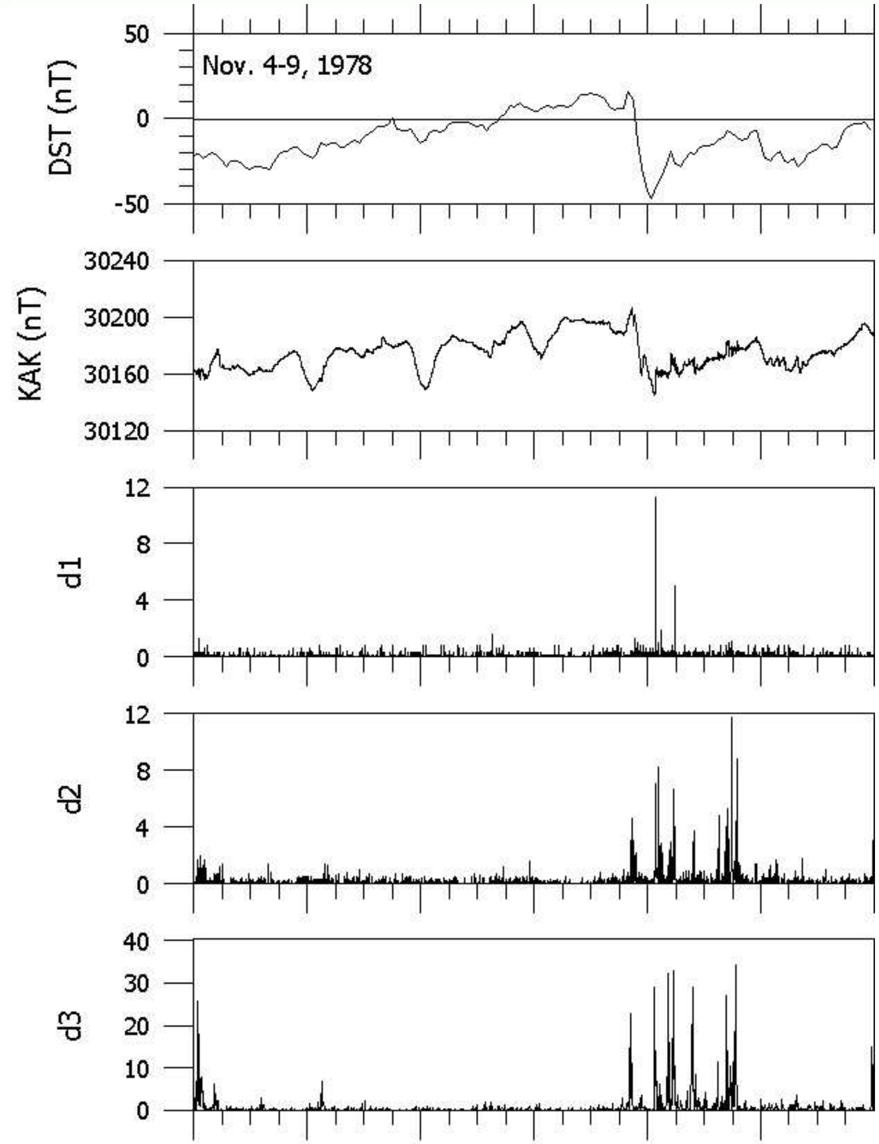
Lawrence & Jarvis(2003)

- studying simultaneous observations of planetary waves from 30 to 220 km
- used the conjugate Fourier transform together with the CWT using Morlet
- this analysis has shown that the relation between planetary waves activity at different altitudes have a high degree of complexity, since there are pulses localized at several altitudes and these series show a non-continuous behavior among them.

Select papers

- geomagnetic minutely signals from Kakioka (Japan)
- for the moderate storm from November 7-8, 1978
- DWT is applied to using Daubechies orthogonal wavelet family 2
- The abrupt variations of the horizontal component of the geomagnetic field are:
  - emphasized by the largest amplitudes of the wavelet coefficients
  - the storm period is detected.

Select papers



Final Remarks

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- From the physical point of view this translates in a certain way, the real meaning of this tool.
- This work is far from being a complete review of this subject in different areas, but it is an attempt to characterize the efforts oriented to the atmospheric wavelet applications, by a selection of some relevant updated published papers.

Final Remarks (cont.)

- To assure a complete understanding and to check the consistency of the obtained results in the investigation of atmospheric phenomena using this technique, a good knowledge of the wavelet tool is required.

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Final Remarks (cont.)

- To assure a complete understanding and to check the consistency of the obtained results in the investigation of atmospheric phenomena using this technique, a good knowledge of the wavelet tool is required.
- There are several wavelet families, as the Meyer wavelet that has been used by Yano et al. 2001, the wavelet packets defined in Coifman et al, 1992 and the multi-wavelet worked out by Lilly Park,1995.
- Yet, in the great majority of atmospheric applications, the Morlet, mexican hat, Haar and Daubechies families of wavelet functions have been extensively used.

Final Remarks (cont.)

- To assure a complete understanding and to check the consistency of the obtained results in the investigation of atmospheric phenomena using this technique, a good knowledge of the wavelet tool is required.
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- Use of wavelet basis as tool to make adaptative numerical solutions in PDE.

Web informantion

Informações	
	www.wavelet.org
	dmsun4.bath.ac.uk/resource/warehouse.htm
	www.uni-stuttgart.de/iag/
	www.cosy.sbg.ac.at/~uhl/wav.html
	norum.homeunix.net/~carl/wavelet/
	ftp.nosc.mil/pub/Shensa/Signal_process/
Softwares	
Amara	www.amara.com/current/wavesoft.html
FracLab/Scilab	www-rocq.inria.fr/scilab/contributions.html
Wavelab	www-stat.stanford.edu/~wavelab/
WaveTresh/R	www.stats.bris.ac.uk/~wavethresh/software
Torrence & Compo	paos.colorado.edu/research/wavelets/software.html

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