Possibility theory, fundamentals and implementation algorithms

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Abstract

Since Aristotle, the truth could be achieved as a consequence of universal method of reasoning by means of which it would be possible to learn everything there is to know about reality. He explains the use of sensation and reason to achieve genuine knowledge. His method of reasoning works with pairs of modes explaining the fact that modal logic deals with the modes of truth. Thus, the *modal logic* introduces the qualification of “necessary” and “possible” premises, but only in a sense that indicates whether the statement is *true* or *false*. The modal logic recognizes the relation and the difference between elements of modal pairs, as follows:

existent	inexistent
possible	impossible
contingent	necessary

But don’t recognizes relations between one element from one pair, with another element in a different pair. This method of reasoning was crated for dealing with the evaluation of truth but which was difficult to interact with certain types of uncertainty.

Recently (in the 70s.), Lotfi Zadeh proposes a new mathematical theory to work friendly with situations where we are strongly impelled to deal with complex types of uncertainty. He first introduced possibility theory in 1978 as an extension of his theory of fuzzy sets and fuzzy logic. D. Dubois and H. Prade further contributed to its development.

Possibility theory says that an event of the real world could be represented by one pair of values, where the possibility and the necessity of the event are evaluated and interpreted in its numerical and/or semantics signification, as follows:

possible	necessary

The possibilistic representation of the uncertainty, witch only requires a linearly ordered scale, is qualitative in nature and provides a faithful representation of partial ignorance. It is new logic approach, able to deal with problems both in case of total or partial ignorance, and still when the variables to consider in the problem are well-known.

In this paper, the interest of possibility theory in the context of an implemented application, where the fundamentals of the theory can be illustrated, mainly the ability to represent the ill-known values by mean of possibility distributions.

As a fuzzy sets and fuzzy logic and even as the imprecise probability theory, the possibility theory is a mathematical theory for dealing with uncertainty, and like those theories possibility works with two basic values. In a fuzzy condition satisfaction problem, a condition is satisfied to a degree (rather than satisfied or not satisfied) and the acceptability of a potential solution becomes a gradual notion.

In the same way, the possibility theory consider two basic values, the possibility *pos()* and the necessity *nec()*, but here, the possibility and the necessity are independent in a real world and don’t represent an interval. One, the possibility, represents the fact that happens, and the other, the necessity represents the repository of knowledge about this kind of fact.

Both values of this pair can be represented, in this project, by an imaginary slider control witch is able to mark values in an interval between 0 and 1. Possibility *pos(0,1)*, and necessity *nec(0,1)*, are able to be valued in their specific and independent conditions, but the condition of possibility can be found in the real world, while the

condition of necessity represents a repository of knowledge about the weight of the attribute in the state of the problem. Thus, the necessity can define the distribution of the possibility of the event to occur.

Combining the two slider controls in well defined stripes (see fig. 1), we are able to establish relations between the real world facts (possibility) and the knowledge involved (necessity) and build faithful semantic outputs and considerations about the problem. An algorithm witch combines a set of attributes that define a problem, and applies a possibilistic distribution of each one, can be solve the problem in a qualitative and semantic way, rather then in a "hard-coded" output solution, also called "crisp" result.

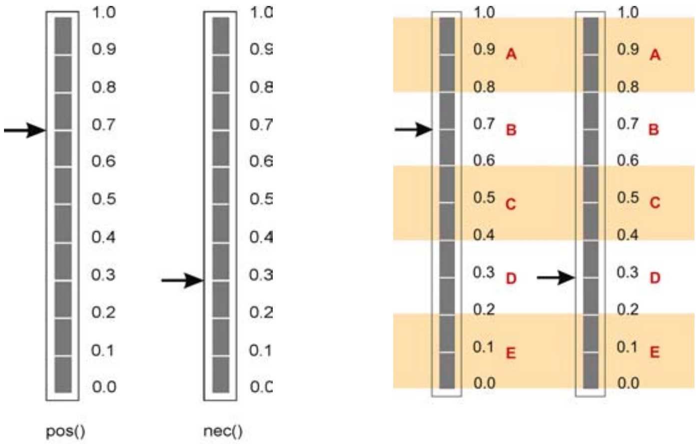

Figure 1

This work presents a sample development of a possibilistic application to diagnose the level of height coronary risks (HCR) in people, based in their variable factors of risk, as the body mass index (BMI), cholesterol index (CI), high blood pressure, smoking habit, sedentary life-style, and others. Basically the idea is apply a possibilistic approach to compares the facts presented in a real situation with the knowledge embedded in a set of linguistic values, able to read the informations and to build semantic analysis and also quantified outputs. The system considered was defined by its specific attributes, represented as a set of sets, as follows:

$$S = \{\{pos(v1), nec(v1)\}, \{pos(v2), nec(v2)\}, \ldots \{pos(vi), nec(vi)\}, \ldots \{pos(vn), nec(vn)\}\}$$

Where v1, v2 ... vn are the variable that define the problem, and each one of them has a value between 0 and 1. v1 can be the value that defines BMI, v2 defines CI, and so one. The application establish 'maps' where the distribution of necessities makes decisions about the semantic outputs related with each attribute, and later, makes intelligent choices about possibilistic forecasts of height coronary risks. The reasoning of the possibilistic logic works like this: 'how much I need of BMI to activate which level of alarm about HCR?' The slider methafor shown in figure 1 presents the level of the attribute from the real world, the possibility (here the real contingence), and apply a critical thinking about its means (role of the necessity), establishing the real significance of the fact. Therefore, we can see that the possibility logic and its careful judgements can be considered a theory of knowledge in a criticist way.

The sample application works with hard numerical values that limits some event, it constraints, as much as ill-known, imprecise, but important constraints, each one restricting the possible significance of values from an specific attribute of an specific case. But the possibilistic algorithm is a realistic approach for the representation of this aspects in a qualitative and semantic way. Thus, when we know several 'crisps' values about some considered universe of discourse, and several ill-known values from the same universe, in spite of this we have the possibility to make careful judgements and take consistent decisions about the problem.

References

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