Artificial Neural Networks Applied to the Lorenz Dynamical System via Variational Data Assimilation

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Abstract

Technique for data assimilation is a crucial issue for the prediction operational process based on a nice combination between data from a mathematical model and observational data. Kalnay (2003) has pointed out that the numerical weather prediction (NWP) is an initial-value problem: given an estimate of the present state of the atmosphere, the model simulates (forecasts) its evolution. The problem of determination of the initial conditions for a prediction model is very important and complex, and it has became into a science itself (Daley, 1991). However, NWP models are obviously not perfect models of the atmosphere, so the use these models to perform predictions is liable to the error. However, there is an observation system, the use this information must be added in order to have a better prediction. Therefore, data assimilation is a method by which the initial conditions for the model are determined from combining of observation data and data from mathematical models. Obviously, as more accurate the estimation of the initial conditions, a better quality of forecasting we will be to have.

Many methods have been developed for data assimilation (Daley, 1991). There are different strategies to combine the forecasting (background, data from the mathematical model) and observations. From mathematical point of view, the assimilation process can be represented by

$$x^a = x^b + W[y^o - H(x^b)] \qquad (1)$$

where x^a is the value of the analysis, x^b is the forecasting (from the mathematical model), W is the weighting matrix, generally computed from the covariance matrix of the prediction errors and observations, y^0 denotes the vector of observations and H represents the observation system.

Data assimilation methods can be placed into two categories. Sequential methods such as the Kalman Filter, which is the optimal solution if the model is linear (which NWP models are not) following a Gaussian statsitics, and variational methods that looks for minimize a cost function (a measure of the misfit between the model and observations over the assimilation period (Roulston, 1998)). The cost function for variational method is given by

$$J(x)=\frac{1}{2}(x^b-x)^T B^{-1}(x^b-x)+\frac{1}{2}\sum_{i=0}^{N}(y^o-H_i(x_i))^T R^{-1}(y^o-H_i(x_i)) \qquad (2)$$

Here the variational schemes (4D-Var) have only an observation term, so minimize a function of the form

$$J(x)=\frac{1}{2}\sum_{i=0}^{N}(y^o-H_i(x_i))^T R^{-1}(y^o-H_i(x_i)) \qquad (3)$$

In this work is showed artificial neural networks (ANN) as a new approach for data assimilation. The performance of multilayer perceptron ANNs is analyzed. The Lorenz system under chaotic regime is used as a test problem, which is given by equation

$$\begin{aligned}\frac{dx}{dt}&=-s(x-y)\\ \frac{dy}{dt}&=rx-y-xz \qquad (4)\\ \frac{dz}{dt}&=xy-bz\end{aligned}$$

For numerical experiments the Lorenz system with parameter $s=10, r=28, b=\frac{8}{3}$ was integrated using a second order Runge Kutta's method, with $\Delta t = 0.001$, and initial conditions $x(0)=-1.5, y(0)=-1.5, z(0)=25$ for the true solution and $x(0)=-1.52, y(0)=-1.5, z(0)=25$ for background solution (a-priori forecast). The data insertion is done at each 12 time-step.

The multilayer perceptron was employed as a implementation of ANN, the analysis is obtained after the MLP-ANN has been trained. The backpropagation algorithm was used for learning phase for emulating analyis from the Kalman filter, and from the variational method. It was considered 2000 prediction data as training data set, and 333 data are used for cross validation. The topology of the MLP consists of one hide layer with two neurons (Härter, 2005).

The goal of this work is to compare data assimilation by the MLP-ANNs emulating both procedures:

the Kalman filter, and variational scheme.

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