DIRECT SIMULATION OF HEAT TRANSFER AND PRESSURE DISTRIBUTIONS OF HYPERSONIC FLOW OVER BACKWARD-FACING STEPS

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Abstract. *This paper describes a numerical study of backward-facing steps situated in a rarefied hypersonic flow by employing the Direct Simulation Monte Carlo (DSMC) method. The work is motivated by the interest in investigating the step-height effect on the aerodynamic surface quantities. The primary aim of this paper is to examine the sensitivity of the heat transfer, pressure and skin friction coefficients with respect to step-height variations. The analysis showed that the hypersonic flow past a backward-facing step is characterized by a strong expansion around the step corner, which influences the aerodynamics surface properties downstream its rear face. it was found that changing the step height affected the aerodynamic surface quantities a distance of a few mean free paths upstream of it. In addition, heat transfer, pressure, and skin friction coefficients were affected downstream the step due to changes on the rear-face height of the steps.*

Keywords: *DSMC, Hypersonic flow, Rarefied flow, Backward-facing step, Aerodynamic heating.*

1. INTRODUCTION

In advanced studies of hypersonic vehicles, a greater understanding of the factors that affect local heating becomes imperative. In the calculation of the heating load, the analysis usually assumes that the vehicle has a smooth surface. Nevertheless, discontinuities such as steps, gaps or cavities are often present in the design of reentry vehicle surfaces. Such surface discontinuities may constitute a potential source of premature transition from laminar to turbulent flow and increased heating. In hypersonic flight, the flow over a step causes locally thermal and aerodynamic loads, which may dramatically exceed the ones of a smooth contour. In order to operate safely detailed knowledge of the heat transfer and pressure field are necessary.

The hypersonic flow past a backward-facing step is characterized by a strong expansion wave around the step corner. Furthermore, flow separation is known to alter the heat transfer at and beyond the separation region. Usually, the low velocity in the recirculation region is expected to cause relatively low heat-transfer rates. Nevertheless, at the reattachment zone, the heat-transfer rates may either increase gradually to their attached flow values or peak to comparatively large values.

Many experimental and theoretical studies (Charwat et al., 1961; Donaldson, 1967; Gai & Milthorpe, 1995; Gai et al., 1989; Grotowsky & Ballmann, 2000; Leite & Santos, 2009; Loth et al, 1992; Rom & Seginer, 1964; Scherberg & Smith, 1967; Shang & Korkegi, 1968) have been conducted in order to understand the physical aspects of a hypersonic flow past to this type of discontinuities, characterized by a sudden change on the surface slope. For the purpose of this introduction, it will be sufficient to describe only a few of these studies.

Rom & Seginer (1964) investigated experimentally the heat-transfer rate on a 2-D backward-facing step in a laminar supersonic flow, corresponding to Mach number in the range of 1.5 to 2.5, and Reynolds number in the range of 10^3 to 10^5. The results indicated that the heat-transfer rates changed with the distance behind the step. In addition, it was found that the heat transfer rates depended on the ratio of the boundary-layer thickness at the separation to the step height.

Data presented by Charwat et al. (1961) indicated that flow which separates from an isolated backward-facing step impinges on the wall approximately a distance of seven times the step height downstream of the step if the boundary layer is laminar, and approximately five times the step height downstream for a turbulent boundary layer.

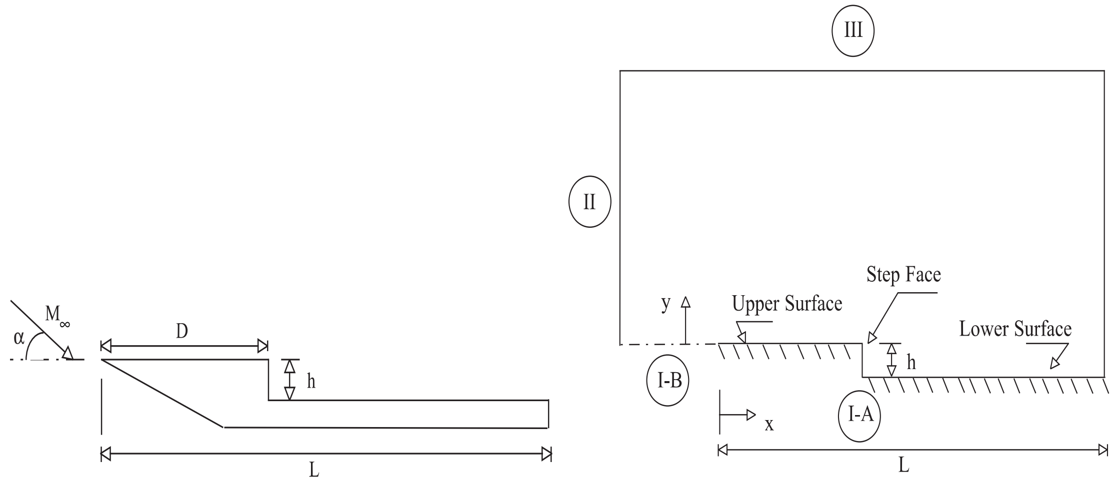
Gai & Milthorpe (1995) presented experimental and computational results of a high enthalpy flow over a blunted-stepped cone. Basically, an axisymmetric backward-facing step of height of 3 mm and 6 mm located at a distance of 101 mm from the nose. The analysis showed that the heat transfer rate was typical of that in a separated flow, i.e., a sudden fall in heat transfer very near the step and then a gradual increase beyond it. The experimental data showed a decrease in heat transfer rate after reattachment, whereas the numerical prediction exhibited a plateau for a considerable distance.

A numerical study on backward-facing steps, situated in a rarefied hypersonic flow, has been examined by Leite & Santos (2009) by employing the DSMC method. The work was motivated by the interest in investigating the step height effect on the flowfield structure. The primary emphasis was to examine the sensitivity of velocity, density, pressure and temperature fields with respect to step-height variations of such backward-facing steps. The analysis showed that the hypersonic flow past a backward-facing step was characterized by a strong expansion wave around de corner of the step, which influenced the downstream separation region. It was found that the recirculation region relies on the rear-face height. The analysis also showed that disturbances downstream the step depended on changes in the rear-face height of the steps.

In continuation of the backward-facing step study, the purpose of the present account is to extend further the previous analysis (Leite & Santos, 2009) by investigating the impact of the rear-face height on the aerodynamic surface quantities. In this scenario, the primary goal of this paper is to assess the sensitivity of the heat transfer, pressure, and skin friction coefficients to variations on the rear-face height of the step. The focus of the present study is the low-density region in the upper atmosphere. At high altitude, and therefore, low density environment, the molecular collision rate is low and the energy exchange occurs under non-equilibrium conditions. As a result, the degree of molecular non-equilibrium is such that the Navier-Stokes equations are inappropriate. In such a circumstance, the Direct Simulation Monte Carlo (DSMC) method will be employed to calculate the hypersonic two-dimensional flow over a backward-facing step.

2. GEOMETRY DEFINITION

In the study being reported herein, discontinuities present on the surface of a reentry capsule are modeled by a backward-facing step. By assuming that the rear face h is much smaller than the nose radius R of a reentry capsule, i.e., $h/R \ll 1$, then, the hypersonic flow over the step may be considered as a hypersonic flow over a flat plate with a backward-facing step. Figure 1(a) illustrates a schematic view of the model employed and presents the important geometric parameters.



(a) Backward-facing step

(b) Computational domain

Figure 1: Drawing illustrating (a) the backward-facing step and (b) the computational domain.

According to Fig. 1(a), M_∞ represents the freestream Mach number, α is the angle of attack, h is the rear-face height, D stands for the location of the step, and L is the total length of the flat plate model. It was assumed that the flat plate is infinitely long but only the length L is considered. It was assumed a rear-face height h of 3, 6 and 9 mm, D/λ_∞ of 50 and L/λ_∞ of 200, where λ_∞ is the freestream mean free path.

An understanding of the rear-face height effect on the aerodynamic surface properties can be gained by comparing the flowfield behavior of flat plates with and without a step. In this manner, a flat plate free of discontinuities, i.e, without steps, works as a benchmark for the cases with steps.

3. COMPUTATIONAL METHOD AND PROCEDURE

In order to study rarefied flow with a significant degree of non-equilibrium, the Direct Simulation Monte Carlo (DSMC) method (Bird, 1994) is usually employed. The DSMC method has become the most common computational technique for modeling complex flows of engineering interest in the transitional flow regime. The DSMC method models a gas flow by using a computer to track the trajectory of simulated particles, where each simulated particle represents a fixed number of real gas particles. The simulated particles are allowed to move and collide, while the computer stores their positions, velocities and other physical properties such as internal energy.

In the present account, molecular collisions are modeled by using the variable hard sphere (VHS) molecular model (Bird, 1981) and the no time counter (NTC) collision sampling technique (Bird, 1989). Energy exchange between kinetic and internal modes is controlled by the Borgnakke-Larsen statistical model (Borgnakke & Larsen, 1975). Simulations are performed using a non-reacting gas model consisting of 76.3% of N_2 and 23.7% of O_2. Energy exchanges between the translational and internal modes, rotational and vibrational, are considered. The probability of an inelastic collision determines the rate at which energy is transferred between the translational and internal modes after an inelastic collision. For a given collision, the probability is defined by the inverse of the number of relaxation, which corresponds to the number of collisions needed, on average, for a molecule to undergo relaxation. The rates of rotational and vibrational relaxation are dictated by collision numbers Z_R and Z_V, respectively. The collision numbers are traditionally given (Bird, 1994) as constants, 5 for rotation and 50 for vibration.

In order to easily account for particle-particle collisions, the flowfield around the backward-facing step is divided into an arbitrary number of regions, which are subdivided into computational cells. The cells are further subdivided into subcells, two subcells/cell in each coordinate direction. The cell provides a convenient reference for the sampling of the macroscopic gas properties, while the collision partners are selected from the same subcell for the establishment of the collision rate. The computational domain used for the calculation is made large enough so that body disturbances do not reach the upstream and side boundaries, where freestream conditions are specified. A schematic view of the computational domain is depicted in Fig. 1(b). According to this figure, side I-A is defined by the body surface. Diffuse reflection with complete thermal accommodation is the condition applied to this side. Side I-B is a plane of symmetry, where all flow gradients normal to the plane are zero. At the molecular level, this plane is equivalent to a specular reflecting boundary. Sides II and III are the freestream side through which simulated molecules enter and exit. Finally, the flow at the downstream outflow boundary, side IV, is predominantly supersonic and vacuum condition is specified (Guo & Liaw, 2001). At this boundary, simulated molecules can only exit.

The numerical accuracy in DSMC method depends on the cell size chosen, on the time step as well as on the number of particles per computational cell. In the DSMC code, the linear dimensions of the cells should be small in comparison with the scale length of the macroscopic flow gradients normal to the streamwise directions, which means that the cell dimensions should be the order of or smaller than the local mean free path (Alexander et al., 1998, 2000). The time step should be chosen to be sufficiently small in comparison with the local mean collision time (Garcia & Wagner, 2000; Hadjiconstantinou, 2000). In general, the total simulation time, discretized into time steps, is based on the physical time of the real flow. Finally, the number of simulated particles has to be large enough to make statistical correlations between particles significant. These effects were investigated in order to determine the number of cells and the number of particles required to achieve grid independent solutions. Grid independence was tested by running the calculations with half and double the number of cells in the coordinate directions compared to a standard grid. Solutions (not shown) were nearly identical for all grids used and were considered fully grid independent.

4. FREESTREAM AND FLOW CONDITIONS

The flow conditions represent those experienced by a capsule at an altitude of 70 km. This altitude is associated with the transitional flow regime, which is characterized by the overall Knudsen number the order of or larger than 10^{-2}. In this fashion, freestream conditions employed in the present calculations are those given by Leite & Santos (2009) and listed in Table 1, and gas properties (Bird, 1994) considered are shown in Table 2. Referring to Tables 1 and 2, T_∞, p_∞, ρ_∞, μ_∞, n_∞, λ_∞, and U_∞ stand, respectively, for temperature, pressure, density, viscosity, number density, molecular mean free path, and velocity, and X, m, d and ω account, respectively, for mass fraction, molecular mass, molecular diameter and viscosity index.

The freestream velocity U_∞, assumed to be constant at 7456 m/s, corresponds to a freestream Mach number M_∞ of 25. The wall temperature T_w is assumed constant at 880 K. This temperature is chosen to be representative of the surface temperature near the stagnation point of a reentry capsule and is assumed to be uniform over the backward-facing step. It is important to mention that the surface temperature is low compared to the stagnation temperature of the air. This assumption seems reasonable since practical surface materials will probably be destroyed

Table 1: Freestream flow conditions

T_∞(K)	p_∞(N/m^2)	ρ_∞(kg/m^3)	μ_∞(Ns/m^2)	n_∞(m^{-3})	λ_∞(m)	U_∞ (m/s)
219.69	5.582	8.753×10^{-5}	1.455×10^{-5}	1.8192×10^{21}	9.285×10^{-4}	7456

Table 2: Gas properties

	X	m (kg)	d (m)	ω
O_2	0.237	5.312×10^{-26}	4.01×10^{-10}	0.77
N_2	0.763	4.650×10^{-26}	4.11×10^{-10}	0.74

if surface temperature is allowed to approach the stagnation temperature.

By assuming the rear-face height h as the characteristic length, the Knudsen number Kn_h corresponds to 0.3095, 0.1548 and 0.1032 for height h of 3, 6 and 9 mm, respectively. Finally, the Reynolds number Re_h, also based on the rear-face height h and on conditions in the undisturbed stream, is around 136, 272, and 409 for height h of 3, 6 and 9 mm, respectively.

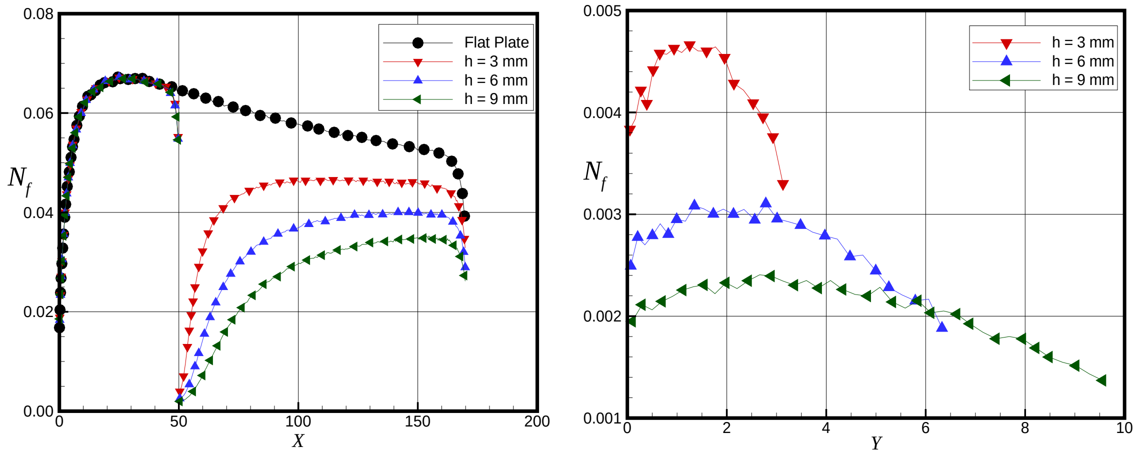
5. COMPUTATIONAL RESULTS AND DISCUSSION

This section focuses on the effects that take place in the aerodynamic surface quantities due to variations on the rear-face height of a backward-facing step. Aerodynamic surface quantities of particular interest in the transition flow regime are number flux, heat transfer, pressure, and skin friction. In this scenario, this section discusses and compares differences in these quantities expressed in dimensionless coefficient form.

5.1 Number Flux

The number flux N is calculated by sampling the molecules impinging on the surface by unit time and unit area. The distribution of the number flux along the step surface – upper, face and lower – is illustrated in Fig. 2 as a function of the step height h. In this group of plots, N_f represents the number flux N normalized by $n_\infty U_\infty$, where n_∞ is the freestream number density and U_∞ is the freestream velocity. In addition, X and Y are the lengths x and y normalized by the freestream mean free path λ_∞. As a basis of comparison, the dimensionless number flux for the flat plate without a step is also illustrated in the plots.

According to Fig. 2(a), it is observed that, from the leading edge up to the vicinity of the step corner, the number flux behavior for the flat plate with a backward-facing step is similar to that one without a step. This is an expected behavior, since the flowfield in this region has no idea about the presence of the step. Nevertheless, very close to the step corner, the number flux drops off due to the flow expansion around the step corner. After that, at the vicinity of the rear face on the lower surface, the number flux suddenly goes to zero and afterwards increases significantly as the flow develops along the lower surface. It should be remarked that, for the flowfield, the step represents a "discontinuity" on the surface. In this manner, the lower the step height h, the faster the number flux approaches the number flux behavior observed for the flat plate without a step.



(a) Upper and lower surfaces

(b) Rear face

Figure 2: Distribution of the number flux N_f along (a) the upper and lower surfaces and along the (b) rear face as function of the step height h.

Referring to Fig. 2(b), it is seen that the maximum number flux to the rear face of the step is an order of magnitude smaller than those values observed for the upper and lower surfaces. This behavior is directly related to the recirculation region downstream of the rear face. It should be mentioned in this context that, the recirculation region at the vicinity of the rear face is a region of low density (Leite & Santos, 2009). As a result, the number flux to the face is very low. Similar to that one showed to the lower surface, the number flux to the face is a function of step height h, i.e, it decreases with the step-height rise.

5.2 Heat Transfer Coefficient

The heat transfer coefficient C_h is defined as follows,

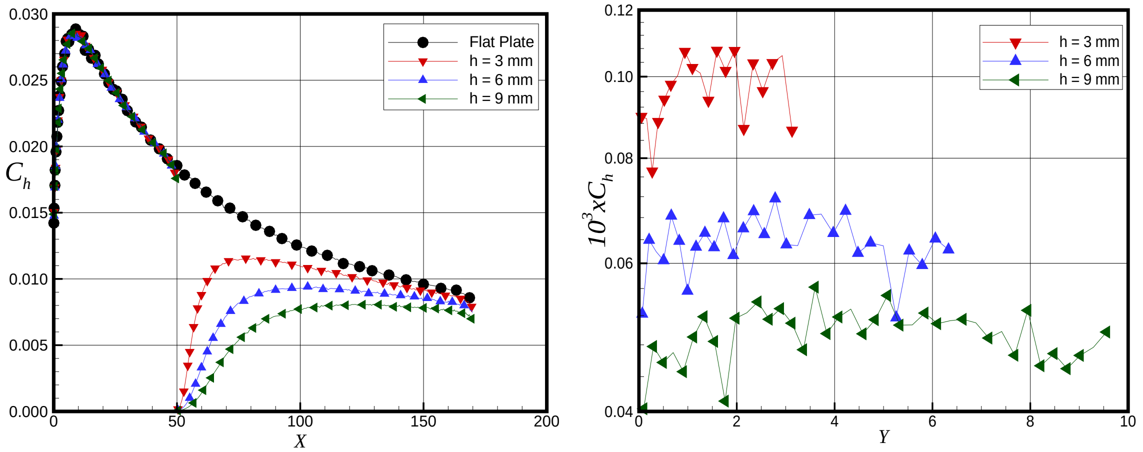
$$C_h = \frac{q_w}{\frac{1}{2}\rho_\infty U_\infty^3} \tag{1}$$

where the heat flux q_w to the body surface is calculated by the net energy flux of the molecules impinging on the surface. A flux is regarded as positive if it is directed toward the body surface. The net heat flux q_w is related to the sum of the translational, rotational and vibrational energies of both incident and reflected molecules, and defined by,

$$q_w = q_i + q_r = \sum_{j=1}^{N}[\frac{1}{2}m_j c_j^2 + e_{Rj} + e_{Vj}]_i + \sum_{j=1}^{N}[\frac{1}{2}m_j c_j^2 + e_{Rj} + e_{Vj}]_r \tag{2}$$

where m is the mass of the molecules, c is the velocity of the molecules, e_R and e_V stand for the rotational and vibrational energies, respectively. Subscripts i and r refer to incident and reflect molecules.

The dependence of the heat transfer coefficient C_h on the rear-face height h is displayed in Figs. 3(a,b) for the upper, face, and lower surfaces. Again, for comparative purposes, the heat



(a) Upper and lower surfaces

(b) Rear face

Figure 3: Distribution of heat transfer coefficient C_h along (a) the upper and lower surfaces and along the (b) rear face as function of the step height h.

transfer coefficient for the flat plate without a step is also illustrated in the plots. According to Fig. 3(a), it is clearly noticed that, except very close to the step corner, the heat transfer coefficient to the upper surface follows the same tendency of that presented by the number flux in the sense that it is not affected by the presence of the step. Along the lower surface, the minimum heat transfer coefficient occurs at the vicinity of the rear face in the recirculation region. After that, the heat transfer coefficient increases to a maximum value after the flow reattachment. In the following, C_h decays and approaches the level observed for the flat plate without a step. It is important to mention that, after the flow reattachment, the flowfield behavior is similar to that along a flat plate. In addition, no shock-wave formation was observed on the lower surface, for the conditions investigated, as shown by Leite & Santos (2009).

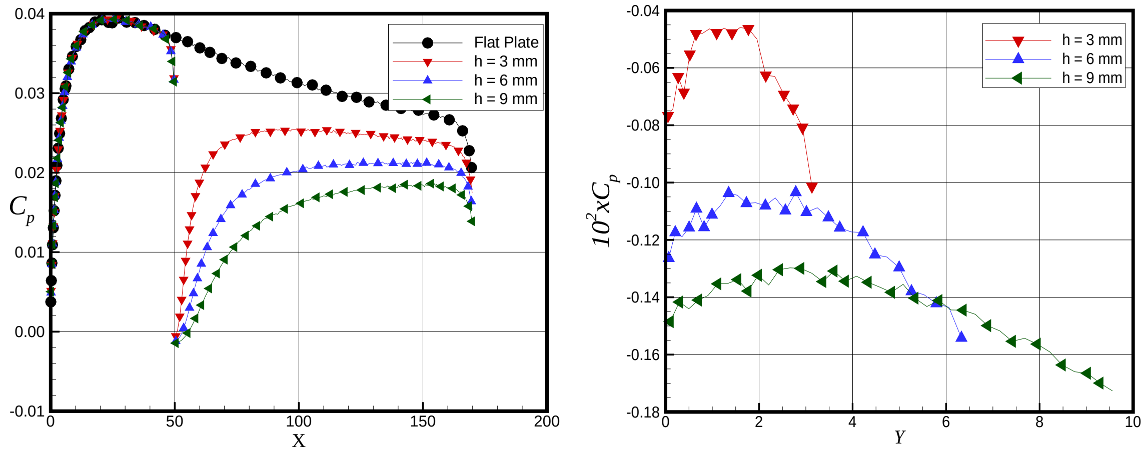
Based on Fig. 3(b), along the rear face, the heat transfer coefficient values are two order of magnitude lower than those observed on the upper or lower surfaces. As mentioned earlier, this is due to the recirculation region formed at the vicinity of the rear face. In this region, density ρ is lower that that of the freestream density ρ_∞, the velocity of the molecules is very low as compared to the freestream velocity, and the temperature is approximately the same as the wall temperature T_w (Leite & Santos, 2009). As a result, there is a balance between the incident heat flux q_i and the reflected heat flux q_r contributions, as defined by Eq. 2, since molecules in this region have low energy.

5.3 Pressure Coefficient

The pressure coefficient C_p is defined as follows,

$$C_p = \frac{p_w - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} \tag{3}$$

where the pressure p_w on the body surface is calculated by the sum of the normal momentum fluxes of both incident and reflected molecules at each time step as follows,



(a) Upper and lower surfaces

(b) Rear face

Figure 4: Distribution of pressure coefficient C_p along (a) the upper and lower surfaces and along the (b) rear face as function of the step height h.

$$p_w = p_i + p_r = \sum_{j=1}^{N} \{[(mv)_j]_i + [(mv)_j]_r\} \tag{4}$$

where v is the velocity component of the molecule j in the surface normal direction.

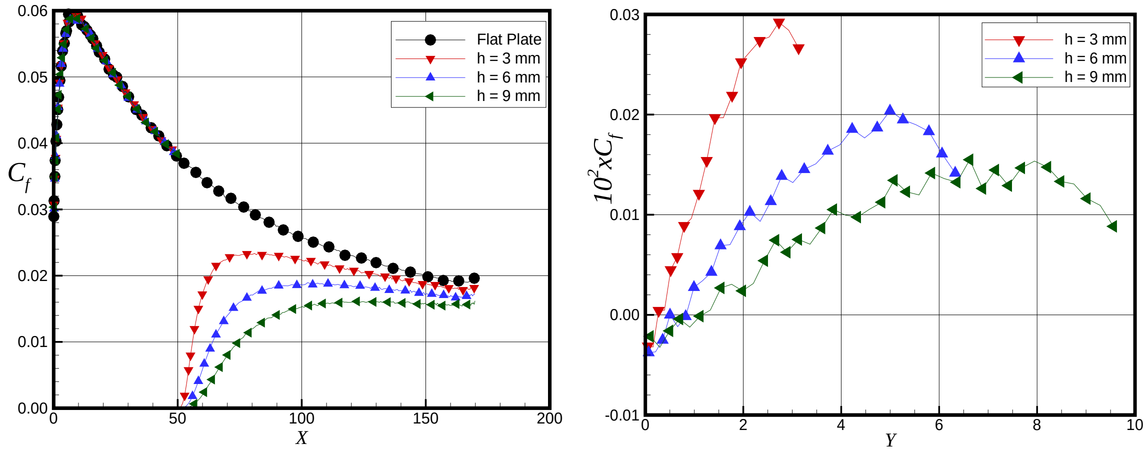
The variation of the pressure coefficient C_p caused by changes in the rear-face height h is demonstrated in Figs. 4(a,b) for upper, face, and lower surfaces. According to these figures, it is clearly noted that the pressure coefficient behavior follows the same trend shown by the number flux in the sense that: (1) from the leading edge up to the the vicinity of the step corner, the pressure coefficient behavior for the flat plate with a backward-facing step is similar to that one without a step, (2) the presence of the backward-facing step affects the pressure coefficient in a small region very close to the step corner, (3) in this small region, the pressure coefficient presents a sudden decrease as compared to the pressure coefficient for the flat plate without step, (4) the smallest values for the pressure coefficient takes place along the rear face of the step, (5) in this region, the wall pressure p_w is lower than the freestream pressure p_∞. Similar to the number flux and the heat transfer coefficient, the smallest values on the pressure coefficient are directly related to the recirculation region, as pointed out earlier.

5.4 Skin Friction Coefficient

The skin friction coefficient C_f is defined as follows,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_\infty U_\infty^2} \tag{5}$$

where the shear stress τ_w on the body surface is calculated by the sum of the tangential momentum fluxes of both incident and reflected molecules impinging on the surface at each time step by the following expression,



(a) Upper and lower surfaces

(b) Rear face

Figure 5: Distribution of skin friction coefficient C_f along (a) the upper and lower surfaces and along the (b) rear face as function of the step height h.

$$\tau_w = \tau_i + \tau_r = \sum_{j=1}^{N} \{[(mu)_j]_i + [(mu)_j]_r\} \tag{6}$$

where u is the velocity component of the molecule j in the surface tangential direction.

It is worthwhile to note that for the special case of diffuse reflection, the gas-surface interaction model adopted herein, the reflected molecules have a tangential moment equal to zero, since the molecules essentially lose, on average, their tangential velocity components. In a diffuse reflection, the molecules are reflected equally in all directions, and the final velocity of the molecules is randomly assigned according to a half-range Maxwellian distribution determined by the wall temperature. In this fashion, the contribution of the reflected tangential momentum flux τ_r that appears in Eq. 6 is equal to zero. Nevertheless, for incomplete surface accommodation, the reflected tangential momentum flux τ_r contributes to the skin friction coefficient.

The influence of the rear-face height in the skin friction coefficient C_f is depicted in Figs. 5(a,b) for upper, face, and lower surfaces. Looking first to Fig. 5(a), it is observed that, in general, the skin friction coefficient behavior is similar to those for the other surface quantities. Along the lower surface, the skin friction coefficient is negative near to the face of the step and becomes positive at the reattachment point. After that, the skin friction coefficient increases and reaches the values observed for the flat plate without steps. The condition C_f = 0 or τ_w = 0, used to define the reattachment point, takes place at X = 50.51, 51.25 e 52.40 for rear-face height h of 3, 6, and 9 mm, respectively. Turning next to Fig. 5(b), it is seen that the skin friction coefficient C_f along the step face is at least two orders of magnitude smaller than that observed along the upper surface.

6. CONCLUDING REMARKS

Computations of a rarefied hypersonic flow on backward-facing steps have been performed by using the Direct Simulation Monte Carlo (DSMC) method. The calculations provided information concerning the nature of the aerodynamic surface quantities on backward-facing steps.

Effects of the rear-face height on the number flux, heat transfer, pressure and skin friction coefficients for a representative range of parameters were investigated. The rear-face height ranged from 3 to 9 mm, which corresponded Knudsen numbers in the transition flow regime.

The analysis showed that the aerodynamic surface properties were affected a distance of a few mean free paths upstream of the step location. It was found that the heat flux to the surface as well as the wall pressure increased along the lower surface downstream the back face. Nevertheless, the heat flux rise and wall pressure rise were below those values for a flat plate without a step, for the conditions investigated in the present account.

7. ACKNOWLEDGEMENTS

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