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MULTIFRACTAL ANALYSIS OF AN INTERMITTENT SOLAR BURST OBSERVED BY THE BRAZILIAN SOLAR SPECTROSCOPE

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ABSTRACT

The solar radio emissions in the decimetric frequency range (above 1 GHz) are very rich in temporal and spectral fine structures due to nonlinear processes occurring in the magnetic structures on the corresponding active regions. In this paper we characterize the singularity spectrum, $f(\alpha)$, for a typical SFU profile coming from solar burst dynamical spectra observed at 1000-2500 MHz. We interpret our findings as evidence of inhomogeneous plasma turbulence driving the underlying plasma emission process and discuss the multifractal approach into the context of BDA data analysis.

INTRODUCTION

Today, several theoretical aspects of solar are related to the high resolution and high sensitivity data observed in the lowest microwave range (1-3 GHz), usually reported as the *decimetric range* (Aschwanden, 2005). It is known that the large amount of energy released during a solar are and the relatively short timescale in which all related events occur lead to the conclusion that a solar are is a magneto-hydrodynamic (MHD) instability taking place in strongly anisotropic turbulent plasma (Kuperus, 1976). The importance of this MHD scenario has been investigated, for example, from nonlinear analysis of decimetric bursts at 3 GHz observed during the June 6, 2000 flare (Rosa et al., 2008). It was found that the 3 GHz radio burst power spectrum exhibits a power-law which is an evidence of stochastic intermittency due to a self-affine dynamics as found in the MHD turbulence theory. Intermittent energetic process implies that the fluctuations are correlated without a dominant characteristic time scale, as predicted in the models for multi-loop interactions (Tajima et al., 1987). However, in order to characterize more precisely the nature of such self-affine turbulent process, a complementary analysis, based on the singularity spectrum technique, is required.

As known from the turbulence theory the intermittency leads to deviation from usual Kolmogorov energy structure functions and its main signature are the singularity spectra exponents,

$f(\alpha)$, which represent a power-law scaling- free dependence (Frisch, 1995). For inhomogeneous plasma turbulence, the so-called multi-fractal *p*-model describes how the energy can be distributed among scales following a multiplicative rescaling structure (Halsey et al., 1986). Although the variability pattern of the 3 GHz burst can be interpreted as a typical profile resulting from a turbulent process, data obtained through Brazilian Decimetric Spectrometer (BSS) (Sawant et al., 2001), in the range of 1-2.5 GHz, are richer in intermittency. Hence, in this paper, taking into account the *p*-model singularity spectra, we obtain the $f(\alpha)$ for a typical BBS burst, improving our search for a robust technique by which inhomogeneous plasma turbulence process might be identified. The importance of the singularity spectra approach is discussed in the context of the Brazilian Decimetric Array (BDA) high resolution data analysis (Sawant et al., 2007).

DATA AND METHODOLOGY

Data

The Brazilian Solar Spectroscope (BSS) is a digital spectroscope and its signal can be recorded up to 100 digital frequency channels. The BSS operates over the frequency range of 1000 - 2500 MHz, with high time (10 - 1000 ms) and frequency (3 MHz) resolutions, in conjunction with the polar mounted 9 meter diameter parabolic antenna. Absolute timing accuracy is 3 ms and the minimum detectable flux is (3 s.f.u. (Madsen et al., 2004).

Figure 1 shows a BSS dynamic spectrum from where we selected the 1890 MHz frequency channel in order to analyze a typical intermittent time series (Figure 2a). This time series was observed with time and frequency resolutions of 100ms and 10 MHz, respectively.

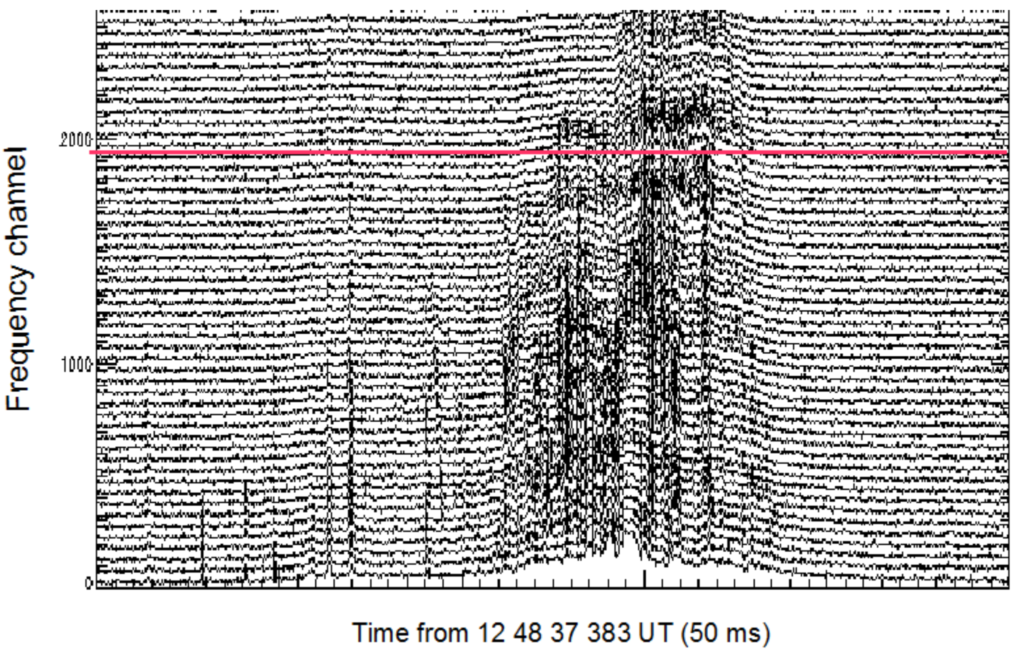


Fig. 1 - The 1000-2500 MHz dynamic spectrum for the solar burst observed from BSS. The red line identifies the 1890 MHz frequency channel.

Multifractal Analysis

Mathematical methods for multifractal analysis of intermittent patterns associated with multiplicative cascades provide a quantitative interpretation of a wide range of physical heterogeneous processes (e.g., Struzik, 2000). Here, we have considered, in order to obtain the singularity spectrum $f(\alpha)$ from the SFU-component time series, the Wavelet Transform Modulus Maxima (WTMM) (Mallat, 1989). The basic idea behind the WTMM method is to describe a partition function over only the modulus maxima of the wavelet transform of a signal SFU(t) (see Appendix). Recently, the robustness of this methodology has been tested for intermittent geomagnetic fluctuations using, as a reference pattern, the so-called *p*-model (Halsey et al., 1986).

In the theory of multifractal statistics the *p*-model is a canonical mathematical system that describes nonhomogeneous energy cascade processes in turbulent flows (e.g., Rodrigues Neto et al., 2001). Hence we performed the theoretical multiplicative cascade *p*-model as a canonical reference in our analysis. The *p*-model is given by

$$\alpha = \frac{\log_2 p_1 + (\omega - 1)\log_2 p_2}{\log_2 l_1 + (\omega - 1)\log_2 l_2} \qquad (1)$$

and

$$f(\alpha) = \frac{(\omega - 1)\log_2(\omega - 1) - \omega\log_2 \omega}{\log_2 l_1 + (\omega - 1)\log_2 l_2} \qquad (2)$$

where (is a free parameter and $l_1 = l_2 = 1/2$ if the eddies are equal as a two-scale Cantor set. In the *p*-model, the largest coherent structure is assumed to be built up by a specific energy flux per unit length and then a scale-independent space-averaged cascade rate occurs. In this process the flux density is transferred to the two smaller eddies with the same length but different flux probabilities p_1 and p_2 ($p_1+p_2 = 1$). The process is repeated several times with p_1 and p_2 randomly distributed, being the asymmetric breakdown in the fragmentation process driven by the parameter $p = p_1 = 1 - p_2$. The common value of p1 = p2 = 0.5 corresponds to the homogeneous energy transfer rate with no intermittency effects. The values of $p > 0.5$ correspond to an intermittent turbulence. In the present work, we use a maximum likelihood algorithm to fit the *p*-model in each characteristic scale.

RESULTS AND INTERPRETATION

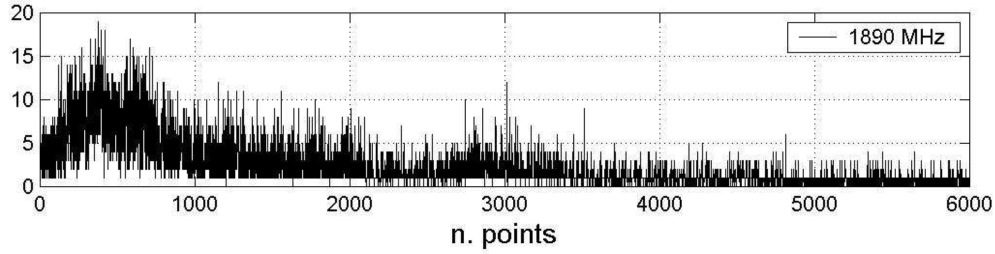
In order to compute the characteristic $f(\alpha)$ for 1890 MHz SFU burst, it is necessary to choose a set of characteristic time scales in the SFU(t) time series to use in the Morlet wavelet transform. Upholding the daily variability component, we use the concept of scale (r) through the difference SFU(t, r) = SFU($t + r$) - SFU(t). As a representative set of MHD oscillations we choose three very close typical characteristics scales: 6, 10 and 15 seconds, observed in our 1.7 min duration time series (Figure 2a). For comparison, we show the respective singularity spectra obtained using the *p*-model. Choosing typical parameters values for the *p*-model (Bolzan et al., 2009) we use an algorithm to fit the experimental and theoretical data. The results are shown in Figure 2b from where the 1890 MHz SFU intermittency can be interpreted as the result of a possibly multifractal process related to the *p*-model nonhomogeneous asymmetric energy cascade. This result makes evident the importance of studying the presence of intermittent phenomena driving the solar decimetric fluctuations.

Table 1 - Singularity spectra parameters for each characteristic scale.

Increment (r)	Time scale (s)	p_1/p_2	l_1	$f_{max,p}$	$f_{max,SFU}$	$\Delta\alpha_p$	$\Delta\alpha_{SFU}$
60	6	0,70	0,23	1,16	1,14	0,67	0,89
100	10	0,75	0,30	1,14	1,15	0,74	0,95
150	15	0,72	0,28	1,16	1,14	0,81	0,99

CONCLUDING REMARKS

The multifractal signature for 1890 MHz decimetric solar burst was successfully detected by using the Wavelet Transform Modulus Maxima and these results are in agreement with multifractal process found for *p*-model inhomogeneous asymmetric energy cascade.



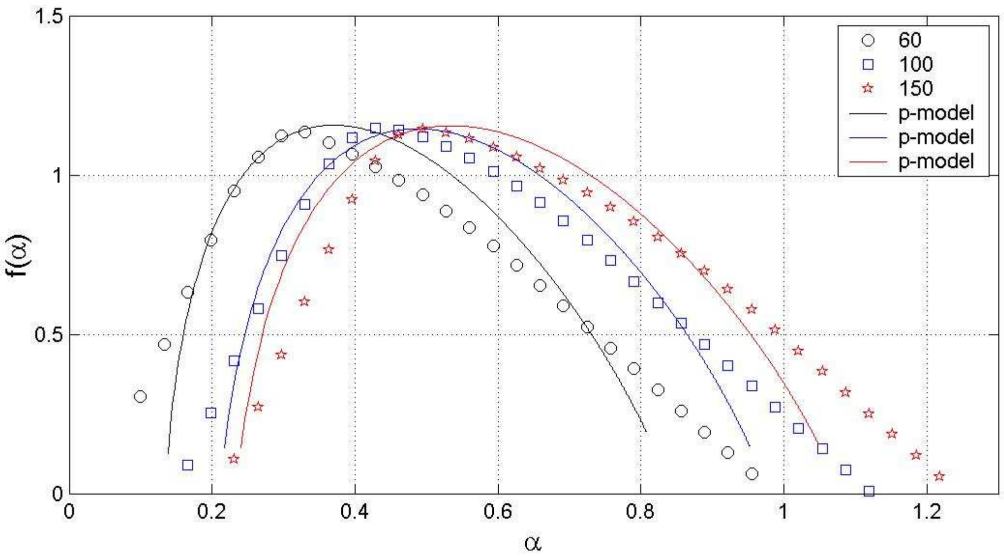


Fig. 2 - (a) The 1890 MHz time profile from the 1000-2500 MHz dynamic spectrum showed in Figure 1. (b) The corresponding singularity spectrum for tree characteristic time scales (60points_100ms = 6s, 10s and 15s) of the 1890 MHz profile and their corresponding *p*-model fitting.

At least for three characteristic scales, 6, 10 and 15 seconds, intermittency is strongly related to multifractal processes where the typical MHD oscillations can play an important role. Thus, 1890 MHz SFU time series can be interpreted as being the response of an out of equilibrium process, possibly related to the particle acceleration from a transversal MHD loop-loop nonlinear interaction and/or to the turbulent interaction between electron beams and evaporation shocks from the optically thick sources of a single loop.

Despite the simplicity of our approach based on a single BSS data set, at least two relevant aspects related to the BDA project are addressed: (i) the need of complementary analysis using a large data set, including the future BDA data and (ii) the need of the high BDA spatial resolution data in order to have images of the related decimetric active regions and their respective energy sources.

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APENDIX

A Singularity Spectrum from Wavelet Transform Modulus Maxima (WTMM)

The wavelet transform of the time series SFU(t) is written as

$$W_\phi\,[SFU(t)](a,b) = \frac{1}{a}\int SFU(t)\varphi^*\left[\frac{t-b}{a}\right]dt, \quad a>0 \qquad \text{(A.1)}$$

where φ^* is the complex conjugate of a continuous wavelet function. This transformation gives the coefficient of the wavelet decomposition of the signal SFU(t) at time t = b for scale a (Enescu et al, 2006). For analysis where the variability pattern contains nonstationary power at many different scales, such as SFU(t), a wavelet analysis based on a plane wave modulated by a Gaussian is required.

Thus, it is then considered the Morlet wavelet, here taken in its form to satisfy the so-called admissibility condition (Farge, 1992)

$$\varphi(t) = \pi^{-1/4}\, e^{i6t}\, e^{-t2/2} \qquad \text{(A.2)}$$

The scaling and translation of this mother wavelet function over the signal SFU(t) are performed by the parameters a and b. While the scale parameter a stretches (or compresses) the mother wavelet to the required resolution, the translation parameter b shifts the basis functions to the desired time location. It can be shown that the wavelet transform can reveal the local characteristics of SFU(t) at a point t_0. More precisely, we have the following power-law relation

$$W_\phi\,[H](a,t_0) \approx |\,s|^{\,\alpha(t0)} \qquad \text{(A.3)}$$

where $\alpha(t_0)$ is the Hlder exponent (or singularity strength). Thus, the exponent $\alpha(t_0)$, for fixed location t0, can be obtained from a log-log plot of the wavelet transform amplitude versus the scale a. However, this power-law characterization is difficult when the process is governed by a hierarchical distribution of singularities compromising the exact determination of α on a finite range of scales. In such case any transformation of the signal SFU(t) may obey some renormalization operation involving multiplicative cascades and it has been demonstrated that the local maxima of $|W\varphi\,(a,b)|$ at a given scale a, are likely to contain all the hierarchical distribution of singularities in the signal.

At a given scale a each one of the WTMM bifurcates into new two maxima giving rise to a rich multiplicative cascade in the limit $a \rightarrow 0$. Thus, it is possible to identify a space-scale partitioning over the maxima distribution and, consequently, a usual *thermodynamical* method of computing the multifractal spectrum of SFU(t) is to define a partition function which scales, in the limit $a \to 0$ as

$$Z(a, q) = \sum_n |W_\varphi H(a, t_n(a))|^q \approx a^{\tau(q)}, \qquad \text{(A.4)}$$

where t_n is the position of all local maxima at a fixed scale s and q is the moment of the measure distributed on the WTMM hierarchy, used to define the power-law scaling of $Z(a,q)$. This power-law yields for small a the scaling exponents $\tau(q)$ - the multifractal spectrum (Muzy et al., 1991). Actually, there is hierarchy of the WTMM that has been used for defining the partition function $Z(a,q)$ based on the multifractal formalism (Arneodo et al., 1995).

The final step in the WTMM method used here is to examine, for a set of scales a, the correspondent singularity spectrum $f(\alpha)$. If one finds a single value _ for all singularities t_n, the signal has a monofractal structure. However, if the underlying process is multifractal, then different parts of the signal are characterized by different values of α (Oswiecimka et al., 2006). The singularity spectrum, approximately an upside-down parabola, peaks at $f_{\alpha,\max}$. The range $\Delta\alpha$ quantifies the fractal non-uniformity, while f_α characterizes how frequently burst components with scaling exponent α occur. The nonhomogeneous turbulent energy cascade is characterized comparing such singularity spectrum parameters for the correspondent turbulent p-model.

REFERENCES

Arneodo, A., Bacry, E., Muzy, J. F., *Physica A*, 213, 1-2, 232-275, 1995.

Aschwanden, A., Springer, Berlin, 2005.

Bolzan, M. J. A., Rosa, R. R., Sahai, Y., Annales Geophysicae 27(2), 569-576, 2009.

Enescu, B., Ito, K., Struzik, Z., *Geophysical Journal International*, 164(1), 63-74, 2006.

Farge, M., *Annual Review of Fluid Mechanics*, 24, 395-457, 1992.

Frisch, U., Cambridge University Press, New York, 1995.

Halsey, T. C., Jensen, M. H., Kadanoff, L. P., Procaccia, I., Shraiman, B. I., *Physical Review A*, 33, 1141, 1986.

Kuperus, M., *Sol. Phys*. 47(1), 7990, 1976.

Madsen, F. R. H., Sawant, H. S., Fernandes, F. C. R., Cecatto, J. R., *Braz. J. Phys.*, 34(4b), 2004.

Mallat, S., *EEE Trans. Pattern Analysis and Machine Intelligence* 11, 674-693, 1989.

Muzy, J. F., Bacry, E., Arneodo A., *Physical Review Letters*, 67, 25, 3515-3518, 1991.

Oswiecimka, P., Kwapien, J., Drozdz, S., *Physical Review E*, 74, 016103, 2006.

Rodrigues Neto, C., Zanandrea, A., Ramos, F. M., Rosa, R. R., Bolzan, M. J. A., S, L. D. A., *Physica A*, 295(1-2), 215-218, 2001.

Rosa, R. R., Karlicky, M., Veronese, T. B., Vijaykumar, N. L., Sawant, H. S., Borgazzi, A. I., Dantas, M. S., Barbosa, E. B. M., Sych, R. A., Mendes, O., *Adv. Space Res.*, 41(5), 844-851, 2008.

Sawant, H. S., Subramanian, K. R., Faria, C., Fernandes, F. C. R., Sobral, J. H. A., Cecatto. J. R., Rosa, R. R., Vats, H. O., Neri, J. A. C. F., Alonso, E. M. B., Mesquita, F. P. V., Portezani, V. A., Martinon, A. R. F., *Solar Phys.*, 200, 167, 2001.

Sawant, H.S. et al., *Solar Phys.*, 242, 213-220, 2007.

Struzik, Z. R., *Fractals* 8, 163-179, 2000.

Tajima, T., Sakai, J., Nakajima, N., Kosugi, T., Brunel, F., Kundu, M. R., *Astrophys. J*, 321, 1031, 1987.