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Stability and Shapes of Premixed Flames in a Cylindrical Burner

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Abstract

We consider in this paper an axially symmetric, angle independent, perturbation Φ of a plane front in a cylindrical burner. We determine the transient states and the constant-speed solutions of the Kuramoto-Sivashinsky equation which describes the behavior of such a flame, analysing the stability of these last solutions. We calculate multiple possible states and transitions of stability which may lead to Hopf bifurcation and, therefore, to time-periodic solutions.

Keywords : Flames, Cellular, Stability, Bifurcation

1 Introduction

Let us consider an axially symmetric, angle independent, perturbation Φ of a plane front in a cylindrical burner of radius \tilde{R} . Let it be \tilde{r} the radial coordinate, $\tilde{\tau}$ time, L the Lewis number of the component of the combustible mixture limiting the reaction, L_0 the critical Lewis number, which depends on the physical properties of the mixture ($L_0 < 1$) and $e = (L_0 - L)/(1 - L_0)$. Then, if the length variables are measured in units of the width l_T of the thermal flame structure and time is measured in units of l_T/U_b - where U_b is the normal velocity of the plane flame front - and the coefficient of gas expansion is assumed equal to one, the evolution of the disturbed flame front is described by the Kuramoto-Sivashinsky equation [1, 2, 3],

$$\frac{\partial \Phi}{\partial \tilde{\tau}} + 4(1 + e)^2 \Delta^2 \Phi + e \Delta \Phi + \frac{1}{2} |\nabla \Phi|^2 = 0. \quad (1)$$

Due to the axial symmetry, one must impose the condition

$$\left. \frac{d\Phi}{d\tilde{r}} \right|_{\tilde{r}=0} = 0. \quad (2)$$

This is not enough to determine a unique solution, hence, in order to fix it, we shall investigate here the case in which

$$\left. \frac{d\Phi}{d\tilde{r}} \right|_{\tilde{r}=\tilde{R}} = 0. \quad (3)$$

In equation (1) Δ is the one-dimension form of the Laplacian operator, i.e.,

$$\Delta = \frac{1}{\tilde{r}} \frac{d}{d\tilde{r}} \left(\tilde{r} \frac{d}{d\tilde{r}} \right).$$

Obviously $\Phi \equiv 0$ is a solution of the boundary value problem (1,2,3); linearizing (1) around it one obtains

$$\frac{\partial \dot{\Phi}}{\partial \tilde{\tau}} + 4(1 + e)^2 \Delta^2 \dot{\Phi} + e \Delta \dot{\Phi} = 0,$$

which has the solution[4]

$$\dot{\Phi}(\tilde{r}, \tilde{\tau}, \lambda) = J_0(\lambda \tilde{r}) \exp(\omega \tilde{\tau}),$$

provided

$$\omega + [4(1 + e)^2 \lambda^2 - e] \lambda^2 = 0.$$

Here, and in the sequel, J_ν stands for the Bessel function of first kind and order ν . The condition (2) is automatically satisfied, while (3) leads to

$$\lambda = \frac{\xi_i}{\tilde{R}},$$

where ξ_i is the i -th zero of J_1 . Thus, the resulting dispersion equation is:

$$\omega + \left[4(1 + e)^2 \left(\frac{\xi_i}{\tilde{R}} \right)^2 - e \right] \left(\frac{\xi_i}{\tilde{R}} \right)^2 = 0 \quad (4)$$

and, therefore, if $e < 0$ the plane front is linearly stable for all modes. Let us then concentrate only on the physically interesting case $e > 0$. With the change of variables

$$\Phi = eu, \quad \tilde{r} = 2e^{-1/2}(1+e)r, \\ \tilde{t} = 4e^{-2}(1+e)^2t,$$

(1) is reduced to the parameter free equation

$$\frac{\partial u}{\partial t} + \Delta^2 u + \Delta u + \frac{1}{2} |\nabla u|^2 = 0. \quad (5)$$

Let R be the new dimensionless radius corresponding to \tilde{R} , and define

$$r = Rz.$$

It is our purpose in this paper to determine constant speed solutions of (5) subjected to conditions (2) and (3) in the form

$$u = -\mu t + f(z),$$

that is, we seek $f(z)$ such that

$$\frac{1}{R^4} \Delta^2 f + \frac{1}{R^2} \Delta f + \frac{1}{2R^2} |\nabla f|^2 = \mu \quad (6)$$

and, if we set $\mu = \frac{\sigma}{R^2}$, $k = 1/R^2$:

$$k\Delta^2 f + \Delta f + \frac{1}{2} |\nabla f|^2 = \sigma, \quad (7)$$

also satisfying

$$f'(0) = 0 \quad (8)$$

$$f'(1) = 0. \quad (9)$$

Going back to the stability problem, the dispersion equation in the new variables is, transposing terms in (4):

$$\omega = -(\xi_i^2 - R^2) \frac{\xi_i^2}{R^4}.$$

In this simpler form, it follows that the stability of the planar solution changes as R goes through the zeros of J_1 . Besides, if $R < \xi_1$, then the plane front is linearly stable to all perturbations of the form $J_0(\xi_i z)$.

2 Analytical solution

It is possible to prove, following alongside a similar proof given by Rabinowitz [5] for a bifurcation theorem, that the boundary value problem (7 - 9)

has non-trivial solutions. For the sake of a simpler notation, we define $\varphi_i(z)$ and α_{rsi} as

$$\varphi_i(z) = \frac{\sqrt{2}}{|J_0(\xi_i)|} J_0(\xi_i z) \quad (J_1(\xi_i) = 0) \quad i \geq 0$$

and

$$\alpha_{rsi} = \int_0^1 z J_1(\xi_r z) J_1(\xi_s z) \varphi_i(z) dz.$$

We shall solve the boundary value problem setting [6, 7, 8]

$$f(z) = \sum_{n=0}^{\infty} f_n(z) \varepsilon^n, \quad f_0(z) = 0, \quad f_1(z) = \varphi_i,$$

$$k = \sum_{n=0}^{\infty} k_n \varepsilon^n, \quad k_0 = 1/\xi_i^2,$$

$$\sigma = \sum_{n=0}^{\infty} \sigma_n \varepsilon^n, \quad \sigma_0 = 0, \quad (10)$$

in (7 - 9). Collecting powers of ε we obtain

$$k_0 \nabla^4 f_n + \nabla^2 f_n \quad (11)$$

$$= \sigma_n - \sum_{j=1}^{n-1} \left\{ k_{n-j} \nabla^4 f_j + \frac{1}{2} \nabla f_j \nabla f_{n-j} \right\},$$

$$f'_n(0) = f'_n(1) = 0. \quad (12)$$

A by-product of the proof of existence of the non-trivial solutions is that $\sum_{n=2}^{\infty} f_n(z) \varepsilon^n$ must be orthogonal to φ_0 , thus

$$\int_0^1 z f_n(z) \varphi_0 dz = \int_0^1 z f_n(z) \varphi_i(z) dz = 0 \quad \forall n \geq 2. \quad (13)$$

Let us now define

$$f_n(z) = \sum_{p=0}^{\infty} \hat{f}_n(p) \varphi_p(z). \quad (14)$$

Utilizing (14) in (11) and defining

$$C_{rs}(z) = \frac{\xi_r \xi_s}{|J_0(\xi_r)| |J_0(\xi_s)|} J_1(\xi_r z) J_1(\xi_s z)$$

we get

$$\sum_{p=1}^{\infty} \left[\left(\frac{\xi_p}{\xi_i} \right)^2 - 1 \right] \xi_p^2 \hat{f}_n(p) \varphi_p(z) = \\ \sigma_n - \sum_{j=1}^{n-1} \left\{ \sum_{p=1}^{\infty} k_{n-j} \xi_p^4 \hat{f}_j(p) \varphi_p(z) \right. \\ \left. + \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) C_{rs}(z) \right\} \quad (15)$$

Conditions (12) are automatically satisfied while (13) imply that

$$\begin{aligned}\hat{f}_n(0) &= 0 \quad \forall n, \\ \hat{f}_n(i) &= \delta_{n1}.\end{aligned}\quad (16)$$

Expanding the products of Bessel functions in (15) we finally obtain

$$\begin{aligned}& \sum_{p=1}^{\infty} \left[\left(\frac{\xi_p}{\xi_i} \right)^2 - 1 \right] \xi_p^2 \hat{f}_n(p) \varphi_p(z) \\ &= \sigma_n - \sum_{p=0}^{\infty} \left\{ \sum_{j=1}^{n-1} \left[k_{n-j} \xi_p^4 \hat{f}_j(p) \right. \right. \\ & \left. \left. + \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) \frac{\xi_r \xi_s \alpha_{rsp}}{|J_0(\xi_r)| |J_0(\xi_s)|} \right] \right\} \varphi_p(z)\end{aligned}\quad (17)$$

and therefore

$$\begin{aligned}\sigma_n &= \sqrt{2} \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) \frac{\xi_r \xi_s \alpha_{rs0}}{|J_0(\xi_r)| |J_0(\xi_s)|} \\ &= \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) D_{rs} \delta_{rs}.\end{aligned}$$

where

$$D_{rs} = \frac{\xi_r \xi_s}{|J_0(\xi_r)| |J_0(\xi_s)|} J_2^2(\xi_r).$$

It follows from the properties of the Bessel functions that

$$\sigma_n = \sum_{j=1}^{n-1} \sum_{r=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(r) \xi_r^2.$$

Choosing now $p = i$ in (17) and using (16)

$$k_{n-1} = \frac{1}{\xi_i^4} \sum_{j=1}^{n-1} \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) \frac{\xi_r \xi_s \alpha_{rsi}}{|J_0(\xi_r)| |J_0(\xi_s)|},$$

and finally, for all other values of p

$$\begin{aligned}\hat{f}_n(p) &= \frac{1}{\left[\left(\frac{\xi_p}{\xi_i} \right)^2 - 1 \right] \xi_p^2} \left\{ \sum_{j=1}^{n-1} \left[k_{n-j} \xi_p^4 \hat{f}_j(p) \right. \right. \\ & \left. \left. + \sum_{r,s=1}^{\infty} \hat{f}_j(r) \hat{f}_{n-j}(s) \frac{\xi_r \xi_s \alpha_{rsp}}{|J_0(\xi_r)| |J_0(\xi_s)|} \right] \right\}\end{aligned}$$

3 Numerical solution

A more direct, although not analytical, solution of (6 - 9) can be obtained by a straightforward expansion of f in terms of Bessel functions[9]. We saw in the previous section that each term of the power series is itself a Fourier-Bessel series, thus,

if we exchange the order of summation and add on the powers of ε for fixed p in (10 - 14), we can write

$$f(z) = \sum_{p=1}^{\infty} \hat{f}_{\infty}(p) \varphi_p(z). \quad (18)$$

The subscript ∞ will have a clear meaning in the next section, when time-dependent functions $\hat{f}(p, t)$ will be introduced for the analysis of the transient states, resulting in $\hat{f}_{\infty}(p) = \lim_{t \rightarrow \infty} \hat{f}(p, t)$. Using (18) in equation (6) we get

$$\begin{aligned}& \sum_{p=1}^{\infty} (\xi_p^2 - R^2) \xi_p^2 \hat{f}_{\infty}(p) \varphi_p(z) = \\ & R^2 \left(\frac{1}{\sqrt{2}} \sigma - \frac{\sqrt{2}}{2} \sum_{r=1}^{\infty} \xi_r^2 \hat{f}_{\infty}^2(r) \right) \varphi_0(z) - \\ & R^2 \sum_{p=1}^{\infty} \left\{ \sum_{r,s=1}^{\infty} \frac{\xi_r \xi_s \hat{u}(r) \hat{u}(s)}{|J_0(\xi_r)| |J_0(\xi_s)|} \alpha_{rsp} \right\} \varphi_p(z).\end{aligned}$$

We thus obtain the following infinite system of nonlinear algebraic equations

$$R^2 \left(\sigma - \sum_{r=1}^{\infty} \xi_r^2 \hat{f}_{\infty}^2(r) \right) = 0 \quad \text{for } p = 0$$

$$\begin{aligned}& (\xi_p^2 - R^2) \xi_p^2 \hat{f}_{\infty}(p) + \\ & R^2 \sum_{r,s=1}^{\infty} \frac{\xi_r \xi_s \hat{f}_{\infty}(r) \hat{f}_{\infty}(s)}{|J_0(\xi_r)| |J_0(\xi_s)|} \alpha_{rsp} = 0 \quad p > 0.\end{aligned}\quad (19)$$

If (19) can be solved, then its solution can be used to determine

$$\sigma = \sum_{r=1}^{\infty} \xi_r^2 \hat{f}_{\infty}^2(r).$$

One must observe that $\hat{f}_{\infty}(p) = 0 \quad \forall p$ is a solution of (19) and that the Jacobian of the system at this solution is the diagonal matrix

$$[(\xi_p^2 - R^2) \xi_p^2 \delta_{pp}].$$

If $R \neq \xi_p \quad \forall p$ then the zero solution is unique. On the other hand if $R = \xi_i$ for some i , the Jacobian is singular. Any numerical method used to solve a finite truncation of (19) will require a starting point, the only simple one being the above mentioned trivial solution. We can then proceed as follows: we first truncate the series (18) to, say, N terms, then employ the algorithm described in the previous section to find an approximate solution of

$$\begin{aligned}& (\xi_p^2 - R^2) \xi_p^2 \hat{f}_{\infty}(p) + \\ & R^2 \sum_{r,s=1}^N \frac{\xi_r \xi_s \hat{f}_{\infty}(r) \hat{f}_{\infty}(s)}{|J_0(\xi_r)| |J_0(\xi_s)|} \alpha_{rsp} = 0\end{aligned}\quad (20)$$

for given R close to ξ_i and $1 \leq p \leq N$. The value of ε corresponding to R is determined recalling from section 2 that

$$\begin{aligned} R &= \frac{1}{\sqrt{k}} \simeq \frac{1}{\sqrt{k_0 + k_1 \varepsilon}} \\ &= \frac{1}{\sqrt{1/\xi_i^2 + k_1 \varepsilon}} = \frac{\xi_i}{\sqrt{1 + \xi_i^2 k_1 \varepsilon}} \\ &\simeq \frac{\xi_i}{1 + \frac{1}{2} \xi_i^2 k_1 \varepsilon} \simeq \xi_i \left(1 - \frac{1}{2} \xi_i^2 k_1 \varepsilon \right), \end{aligned}$$

hence, $\varepsilon \simeq 2(\xi_i - R)/\xi_i^2 k$. Finally, use the approximation thus obtained as the initial guess for the numerical method.

4 Transient states and stability

Observe that if we substitute $\hat{f}_\infty(p)$ by $\hat{f}(p, t)$ in (18) and add from $p = 0$, then we can use the resulting function $u(z, t)$ for determining the transient states of the solutions of equation (5). Doing so, we obtain

$$\hat{f}'(0, t) = -\frac{1}{R^2} \sum_{j=1}^{\infty} \xi_j^2 \hat{f}^2(j, t), \quad (21)$$

$$\begin{aligned} \hat{f}'(i, t) &= -\frac{1}{R^2} \left\{ \xi_i^2 \left[\left(\frac{\xi_i^2}{R^2} \right) - 1 \right] \hat{f}(i, t) \right. \\ &\left. + \sum_{p, q=1}^{\infty} \frac{\xi_p \xi_q \alpha_{pqi}}{|J_0(\xi_p) J_0(\xi_q)|} \hat{f}(p, t) \hat{f}(q, t) \right\}. \end{aligned} \quad (22)$$

One must observe that (22) is a closed system, that once solved, can be used for the integration of (21). As before, for practical purposes, we shall deal with truncations of f with N terms, referring to the resulting systems also as (21) and (22), being clear from the context whether it is a truncation or the full system.

Let us consider an initial perturbation of the form $f(0) = \varepsilon \varphi_m(z)$, we then have

$$\hat{f}'(0, 0) = -\frac{1}{R^2} \xi_m^2 \hat{f}^2(m, 0) = \frac{\varepsilon^2}{R^2} \xi_m^2,$$

and

$$\begin{aligned} \hat{f}'(i, 0) &= -\frac{\varepsilon}{R^2} \left\{ \xi_m^2 \left[\left(\frac{\xi_m}{R} \right)^2 - 1 \right] \delta_{im} \right. \\ &\left. + \frac{\xi_m^2 \alpha_{mmi}}{J_0^2(\xi_m)} \varepsilon \right\}, \end{aligned} \quad (23)$$

and it follows that it does not matter whether the mode is linearly stable or unstable, it will become different from zero for $0 < t \ll 1$. Furthermore, still for t small, one can expect $f(i, t)$, $i \neq m$, to be of order ε^2 while $f(m, t) = O(\varepsilon)$. As t increases, the behavior of the solutions is determined by those modes with the greater rates of growth

$$\sigma_j = -\frac{\xi_j^2}{R^2} \left[\left(\frac{\xi_j}{R} \right)^2 - 1 \right].$$

One must observe that these rates are uniformly bounded above by $3/16$. In fact, as R increases from 0 to ∞ , σ_j goes from $-\infty$, through zero at $R = \xi_j$, reaches its maximum of $3/16$ at $R = 2\xi_j$ decreasing again to zero. This is an important property, one must recall that $\xi_j \simeq j\pi$. Then for any n , $\xi_{2n} \simeq 2n\pi \simeq 2\xi_n$ which means that at the onset of the instability of the $2n$ -th mode, the n -th one attains its maximum rate of growth; however, their importance is inverted again for R not much greater than ξ_{2n} . In fact, for any pair j, l , $\sigma_j = \sigma_l$ implies

$$\xi_j^4 - R^2 \xi_j^2 = \xi_l^4 - R^2 \xi_l^2.$$

Therefore

$$\xi_j^4 - \xi_l^4 = R^2 (\xi_j^2 - \xi_l^2),$$

and then

$$R_{j,l} = \sqrt{\xi_j^2 + \xi_l^2}$$

with

$$\sigma_j = \sigma_l = \frac{\xi_j^2 \xi_l^2}{(\xi_j^2 + \xi_l^2)^2}.$$

In particular, for $j = n, l = 2n$ we have:

$$\begin{aligned} R_{n,2n} &= \sqrt{\xi_n^2 + \xi_{2n}^2} \\ &= \xi_{2n} \sqrt{1 + \frac{\xi_n^2}{\xi_{2n}^2}} \\ &\simeq \xi_{2n} \sqrt{1 + \frac{1}{4}} \simeq 1.12 \xi_{2n}. \end{aligned}$$

Thus, as long as $0.12 \xi_{2n} < \pi$, we have that in a right neighborhood of ξ_{2n} the leading mode is the n -th one. This phenomenon can not take place at odd ordered eigenvalues. At, say, ξ_{2n+1} we have that the n -th rate of growth is already decreasing.

This dominance of certain modes, which depends on the value of R , leads to a balance among the different terms of equation (22), with the consequence of a bounded behavior of the solution, resembling, as it should be, the solution of the Landau equation. Figures 1,2,3 show $f(1, t)$ and $f(2, t)$ and the

corresponding flame velocity $f(0, t)$ obtained by numerical integration of the system using Romberg's method with $N = 20$ for $R = 5$ and $0 \leq t \leq 100$. It is evident from them that $f(0, t)$ approaches a linear dependence on t as the time increases, while $f(1, t)$ and $f(2, t)$ converge to constants. This naturally suggests the use of equation (22) for the linear stability analysis of the constant-speed solutions $\hat{f}_\infty(p)$ obtained in the previous section, because they are also solutions of (22). In fact, if one linearizes this system at $\hat{f}(p, t) = \hat{f}_\infty(p)$, $p \geq 1$, one obtains a linear system of ordinary differential equations whose matrix is minus the Jacobian matrix used for the determination of $\hat{f}_\infty(p)$, i.e., its eigenvalues determine whether this solution is stable or not.

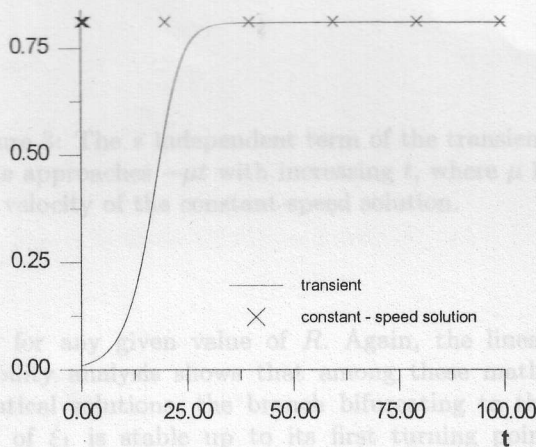


Figure 1: Transient state of the Fourier-Bessel coefficient $\hat{f}(1, t)$. It presents a typical behavior of a solution of the Landau equation. As $t \rightarrow \infty$ $\hat{f}(1, t) \rightarrow \hat{f}_\infty(1)$, indicated with the symbol \times .

5 Discussion of results and conclusions

As described in section 3 we used the algorithm of section 2 with an order of approximation equal to 20 to determine the Fourier-Bessel coefficients of the solutions and the coefficients of the Maclaurin's series of k and σ at the eigenvalues ξ_i for $1 \leq i \leq 6$, employed Horner's summation method to determine the initial guess for the system (20) with $N = 20$ and solved it utilizing Newton-Raphson's method [10].

Figures 4 through 9 show the dependence of the velocity μ and the coefficients $\hat{f}_\infty(p)$ $1 \leq p \leq 5$ on the radius R . Several features are worth to be

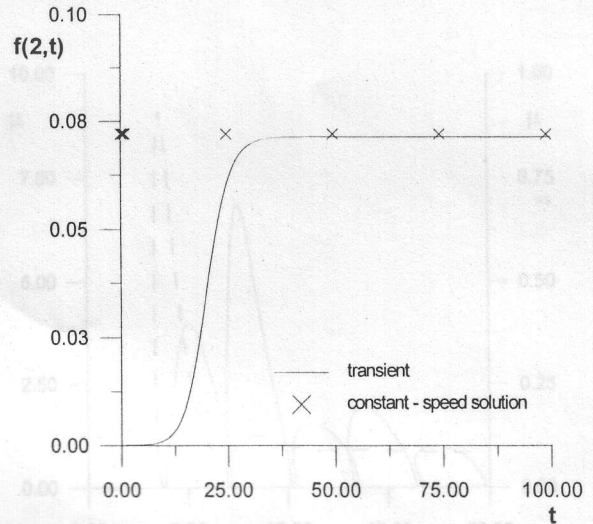


Figure 2: Same as Figure 1 with $\hat{f}(2, t)$ instead of $\hat{f}(1, t)$

observed in them: contrary to what happens with the one-dimension problem, which has periodic solutions for $k \leq 1$ [6], 1 being the first eigenvalue, counted from right to left, where the solution bifurcates subcritically, here the bifurcation at ξ_1 is transcritical, existing a non-trivial solution for $R < \xi_1$. Furthermore, this solution exists up to the minimum value of R , $R_{1 \min} \simeq 3.366706$, with a corresponding flame velocity $\mu_{1 \min} \simeq 3.070748$. We have seen that the planar front is linearly stable for $R < \xi_1$, while of the two solutions which exist in the interval $R_{1 \min} \leq R \leq \xi_1$, the linear stability analysis indicated in section 4 shows that the one with smaller absolute value is unstable. In fact, the branch bifurcating to the left of ξ_1 is unstable up to $R_{1 \min}$, becoming stable at the turning point. Thus, only the former is not physically realizable. This can probably be explained in terms of the mass flux and curvature. The stable flame corresponds to one with a greater velocity and positive curvature at the center; it is well known that for a Lewis number L smaller than one [11] the stability is determined by two main parameters: one is the balance between the incoming mass flux - normal to the flame front - received by the concave and convex regions of the flame front; the other is the heat loss to the fresh gas. One can thus expect that the greater velocity yields a larger mass flux, enough to sustain the flame, while the other one dies toward the planar front.

Another interesting point is the multiplicity of solutions, not just in the interval $R_{\min} \leq R \leq \xi_1$,

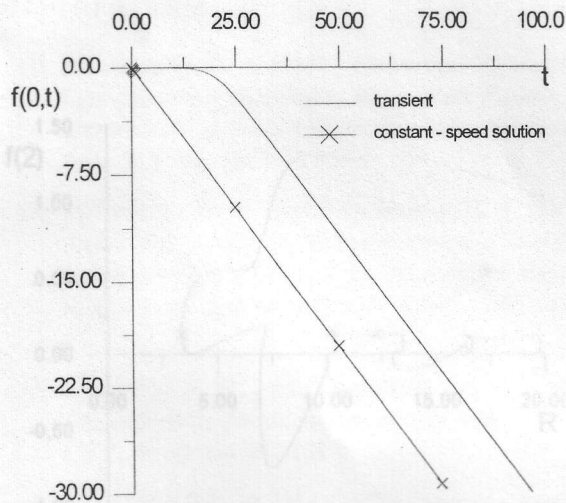


Figure 3: The z independent term of the transient state approaches $-\mu t$ with increasing t , where μ is the velocity of the constant-speed solution.

but for any given value of R . Again, the linear stability analysis shows that among these mathematical solutions, the branch bifurcating to the left of ξ_1 is stable up to its first turning point $R \simeq 7.195$ (passing ξ_2) regaining stability in the interval $10.233 \leq R \leq 12.266$. Two observations are in order: the first one is that the leftmost stable section has $\hat{f}_\infty(1) > 0$ implying a negative curvature at the center of the flame, that is, as long as the radius is not very large and the interaction of the modes moderate, the incoming mass flux is enough to sustain the flame. The second observation is, perhaps, even more important. The transition of the stability at the right bound of the interval indicates a possible Hopf bifurcation[12], and therefore, this may lead to oscillating flames

We can thus conclude that it is possible to have corrugated cylindrical flames for a radius $R \geq R_{\min}$. For lesser values of the radius, the flame would be so small as to seem plane anyway. Multiple states are also possible, however for limited values of the radius only.

Acknowledgments

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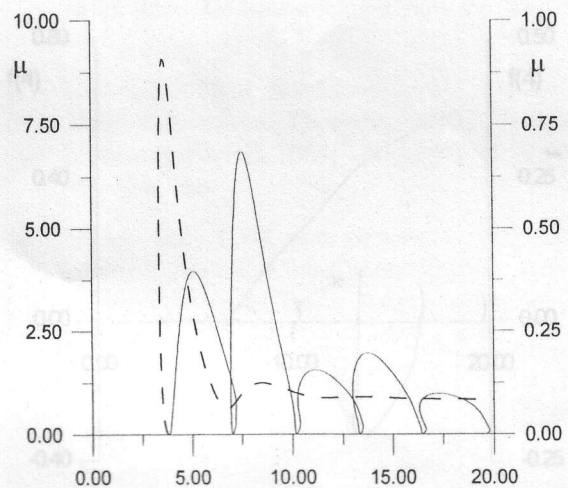


Figure 4: Dependence of the flame velocity μ on the radius R . The left scale corresponds to the branch, indicated with a darker line, that bifurcates to the left of ξ_1 .

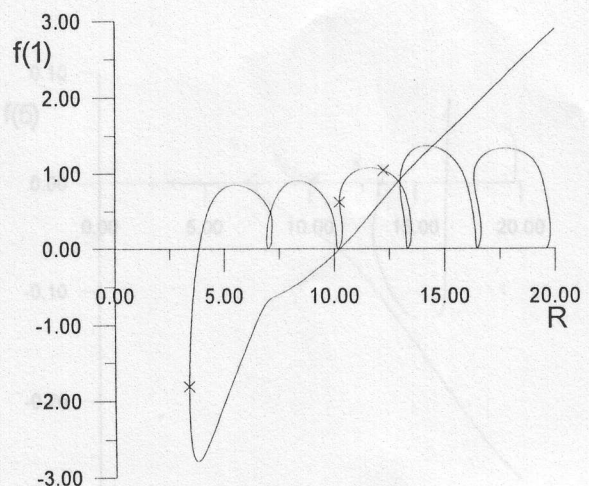


Figure 5: Dependence of the Fourier-Bessel coefficient $\hat{f}_\infty(1)$ on the radius R . The points of transition of stability are indicated with the symbol \times

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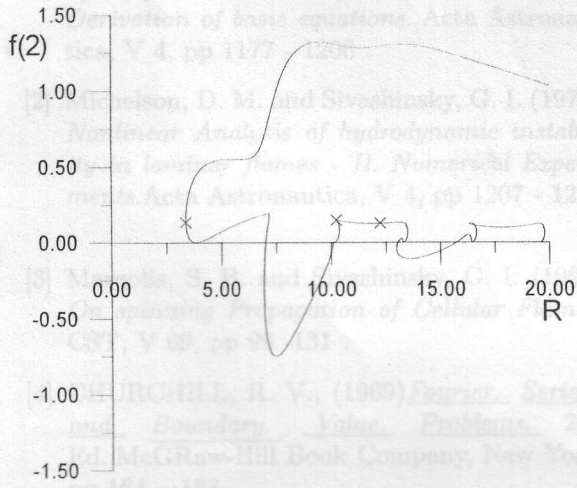


Figure 6: Same as Figure 5 with $\hat{f}_\infty(2)$.

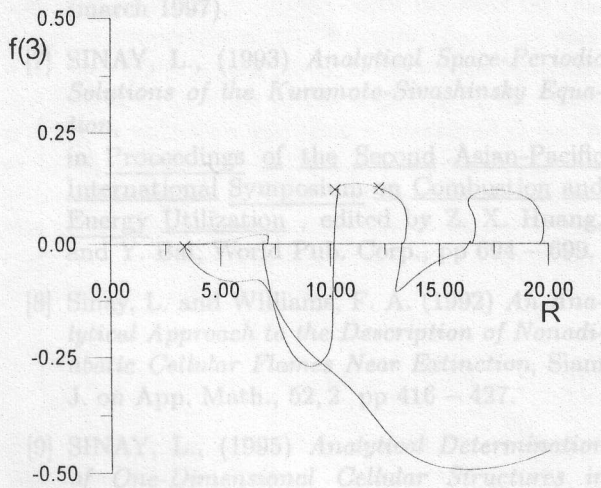


Figure 7: Same as Figure 5 with $\hat{f}_\infty(3)$.

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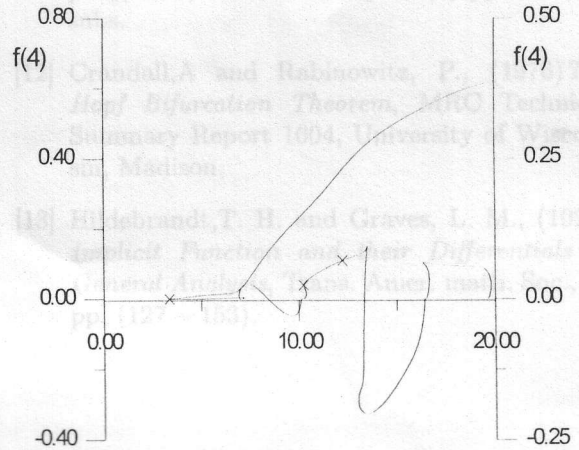


Figure 8: Same as previous figures. Notice the difference in scales, the left ordinates axis corresponds to the branch bifurcating to the left of ξ_1 , and which reaches the greatest positive value at $R = 20$.

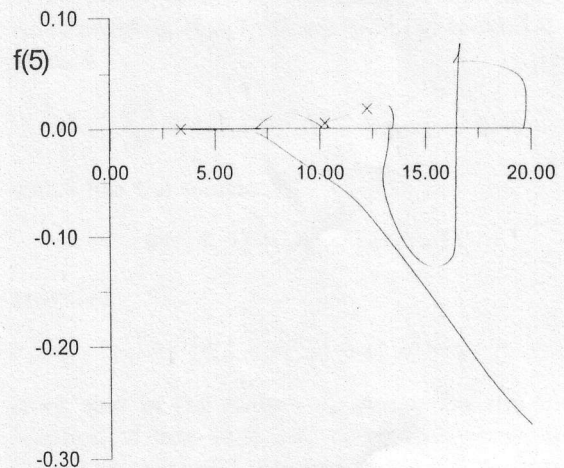


Figure 9: Same as previous figures.

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