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The Zero Relative Radial Accelerations along a family of periodic orbits around L4 and L5 in the Earth-Moon System

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Abstract. *Assume a constellations of satellites flighting close a given nominal trajectory around L4 or L5 in the Earth-Moon system in such a way that there is freedom in the selection of the geometry of the constellation. We are interested in to avoid large variations of the mutual distance between the spacecrafts. In this case, the possible existence of regions of zero relative radial acceleration with respect to the nominal trajectory will prevent from the expansion or contraction of the constellation. The goal of this paper is the study of theses regions.*

Key words: *Constellations of satellites, Zero relative radial acceleration, Earth-Moon.*

1. Introduction

The present paper is devoted to study the existence of regions of zero relative radial accelerations with respect to a family of periodic orbits around L4 or L5 in the Earth-Moon system for formation flying. All the work is done in the force model defined by the Circular Restricted Three Body Problem (CRTBP). The concrete goals of the paper are:

- The study of geometries, around arbitrary periodic orbits with good properties for formation flight.
- The study of the zero relative radial accelerations regions obtained from the preceding analysis.

As reference solutions, along which we will assume that the formation is moving, we will use a family of periodic orbits around L4 and compare the results in each case.

2. The Zero Relative Radial Accelerations Lines

In order to avoid expansion or contraction in a constellation of spacecrafts, we have studied the existence of regions with zero relative radial acceleration (ZRRA) [1]. For a simple model, such as the CRTBP, it is possible to compute an analytical expressions for the above regions if the radius of the constellation (largest separation between spacecrafts) is small, such that a linear approach gives the relevant information about the local dynamics of the problem. For simplicity, our model only considered motions in the plane of the orbit of the Earth-Moon system. Considering the sum of the masses of the two primaries, the distance between them and also making the Newton's gravitational constant equal the unity, in a frame rotating with the primaries, the equations of the planar motion for the spacecraft are [2]:

$$\begin{aligned}
\dot{x} &= u \\
\dot{y} &= v \\
\dot{u} &= x + 2v - (1-\mu)\frac{x+\mu}{\rho_1^3} - \mu\frac{x-1+\mu}{\rho_2^3} \\
\dot{v} &= y - 2u - \left(\frac{1-\mu}{\rho_1^3} + \frac{\mu}{\rho_2^3}\right)
\end{aligned} \tag{1}$$

where $\rho_1^2 = (x+\mu)^2 + y^2$ and $\rho_2^2 = [x-(1-\mu)]^2 + y^2$, and μ denotes the ratio between the Moon mass and sum of the Earth and the Moon masses, $\mu = 0.01215$ [3]. In this system of differential equations, the conversion factor between the velocities is equal to $1.024\ \mathrm{km/s}$ and the normalized system is obtained considering the unities of length and time equal to $384,405\ \mathrm{km}$ (average distance between the Earth and the Moon) and 104.361 hours (Moon's orbital period, 27.322 days, is equal to 2π) [4], respectively.

Denoting the above system by $\dot{X} = f(X(t))$, the linear behaviour around a periodic solution $X_h(t)$ is given by (see Figure 1)

$$\Delta\dot{X}(t) = Df(X_h(t))(t) \tag{2}$$

where

$$Df = \left(\begin{array}{cc|cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline f_x^3 & f_y^4 & 0 & 2 \\ f_x^4 & f_y^3 & -2 & 0 \end{array}\right)$$

and f^3 and f^4 are the last two components of the vector-field f, of which we have to compute the their partial derivatives with respect to x and y in order to get the symmetric sub-matrix F. Writing the array as $(r, \dot{r})^T$, where $r = (\Delta x, \Delta y)$, the linear system (2) becomes

$$\begin{pmatrix} \dot{r} \\ \ddot{r} \end{pmatrix} = \begin{pmatrix} 0 & I \\ F & J \end{pmatrix} \begin{pmatrix} r \\ \dot{r} \end{pmatrix} \tag{3}$$

The points with zero relative velocity are those such that $\dot{r} = 0$, and, in this case, we have that the relative acceleration is given by

$$\ddot{r} = Fr$$

Therefore, the radial component of the relative acceleration will be zero in the set of points where the vectors $\ddot{r}$ and $\dot{r}$ are perpendicular, in other words,

$$r^T F r = 0 \tag{4}$$

Equation (4) represents two lines which depend on the point $X_h(t)$ selected along the periodic solution of (1) as shown in Figure 2.

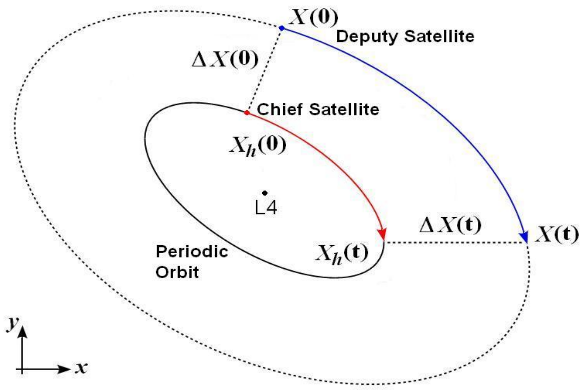

Figure 1. Illustration of a solution $X(t)$ along a periodic solution $X_h(t)$ around $L4$

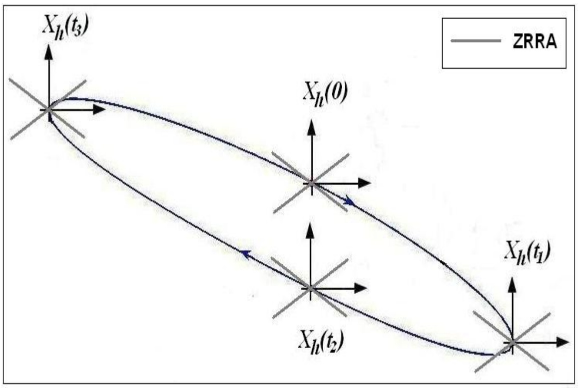

Figure 2. Illustration of Zero relative radial acceleration lines along a periodic solution

The Zero Relative Acceleration Lines (ZRRAL) can be also computed numerically. Given a certain periodic orbit, we select a point on it $(x_h(t), v_h(t))$. Around this point we consider a sphere, in the configuration space, of radius δ and we set the velocity of all the points of the sphere equal to the velocity of the point selected, $v_h(t)$ (zero relative velocity condition). Using polar coordinates, the set of points of the sphere will be the form (see Figure 3).

$$(x_h(t) + s(\theta), v_h(t)).$$

Now, writing the equations of the motion of CRTBP as

$$\ddot{x} = g(x, \dot{x}),$$

the relative acceleration can be evaluated by

$$a(t,\theta) \;=\; g(x_h(t), v_h(t)) - g(x_h(t) + s(\theta), v_h(t)),$$

whose scalar product with $s(\theta)$ will be the desired relative radial acceleration for each angle θ. The points in the sphere such that the scalar product is equal to zero belong to ZRRAL.

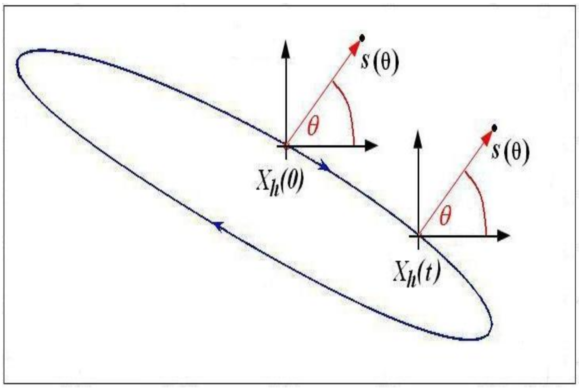

Figure 3. Illustration of a sphere of radius δ for each point of the periodic solution $X_h(t)$

3. ZRRA in Long and Short Period Family

In this section we are interested in to determine the existence of the ZRRAL along periodic orbits around L4. The existence of the ZRRAL along any nominal trajectory is determined by the sign of the discriminant of the sub-matrix F in Equation (4) which represents, in general, a quadratic. If the discriminant of F is negative at a certain point of the nominal trajectory, therefore the ZRRAL at this point is represented by an ellipse of radius zero, i.e. there is no region with ZRRA. Otherwise, the ZRRA at this point is represented by two lines (in the planar case).

Figure 4 shows two periodic orbits around $L4$. The orbit that belongs to the long period family has a period of approximately 91 days. Similarly, the orbit that belongs to the short period family has a period of approximately 29 days. Both are inside of a sphere of a radius of $10,000$ km centered in $L4$.

The value of the discriminant of the sub-matrix F associated to the points along the previous long and short period orbits is shown in Figures 5(a) and 5(b), respectively. As we can see, the sign of the discriminant in both trajectories are always negative, this means that there are no regions with ZRRA along these specific periodic orbits.

The previous fact can be verified when we compute numerically the ZRRA at some points on these periodic orbits. In this manner, considering a sphere of radius equal to 1.0 km, the first row of Figure 6 shows the scalar product between the relative acceleration $a(t,\theta)$ and vector $s(\theta)$ as a function of angle θ at three different points along the long period orbit, i.e. $t = 22, 45$, and 68 days, respectively. Similarly, the second row of Figure 6 shows the scalar product at three different points along the short period orbit, i.e. $t = 7, 14$, and 22 days, respectively.

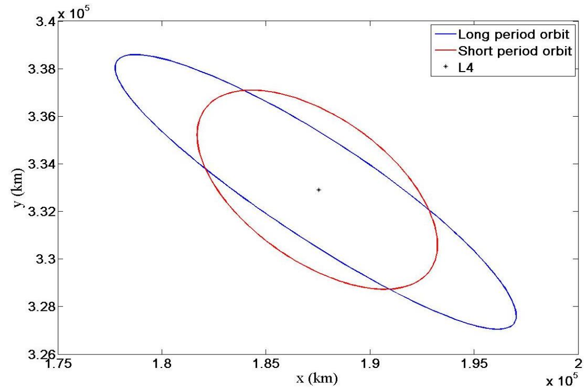

Figure 4. Long and short period orbits around $L4$.

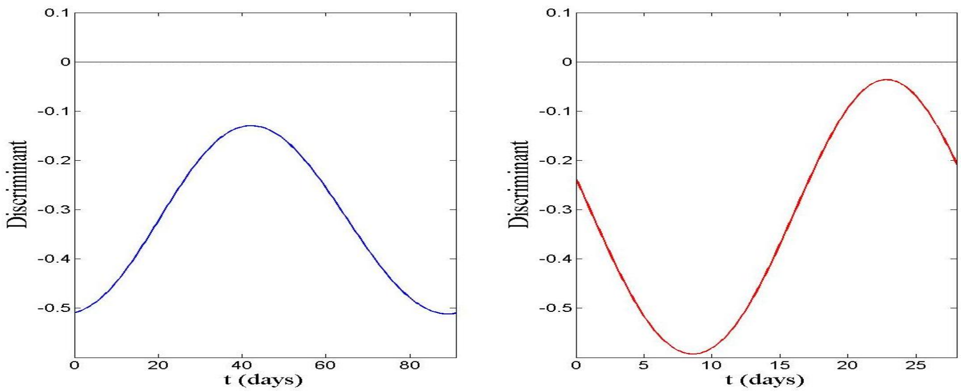

Figure 5. Discriminant of the sub-matrix F associated to the points along the previus: (a)long and (b)short period orbits

As we can see in Figure 7, the scalar product function between the relative acceleration and relative position vectors never crosses the horizontal axis for any angle θ at each of these three points along the previous periodic orbits. Therefore, at each of these three points, the radial component of the relative acceleration $a\left(t,\theta\right)$ is different from zero for any angle θ.

The qualitative behaviour of the function $a\left(t,\theta\right)$ is almost the same for all the values of t if we move along the previous periodic orbits. There appear two maxima and two minima. This behaviour can be seen in the first and second rows of Figure 8 which shows the function θ related to the maximum (left) and minimum (right) for all the values of t along previous long and short period orbits, respectively.

In this manner, from this first example, we can affirm that there is no regions with ZRRA along the periodic orbits that are close enough to $L4$. The next step, therefore, is to explore the existence of regions with ZRRA along periodic orbits that are farther from $L4$.

Figure 9 shows two periodic orbits around $L4$, which have a period of 92 and 29 days, respectively. Both are farther from $L4$ with respect to the periodic orbits shown in Figure 4.

Similarly, Figures 10(a) and 10(b) show the value of the discriminant of the sub-matrix F associated to the points along the previous long and short period orbits, respectively. Unlike the previous case, the sign of the discriminant in both trajectories are negative

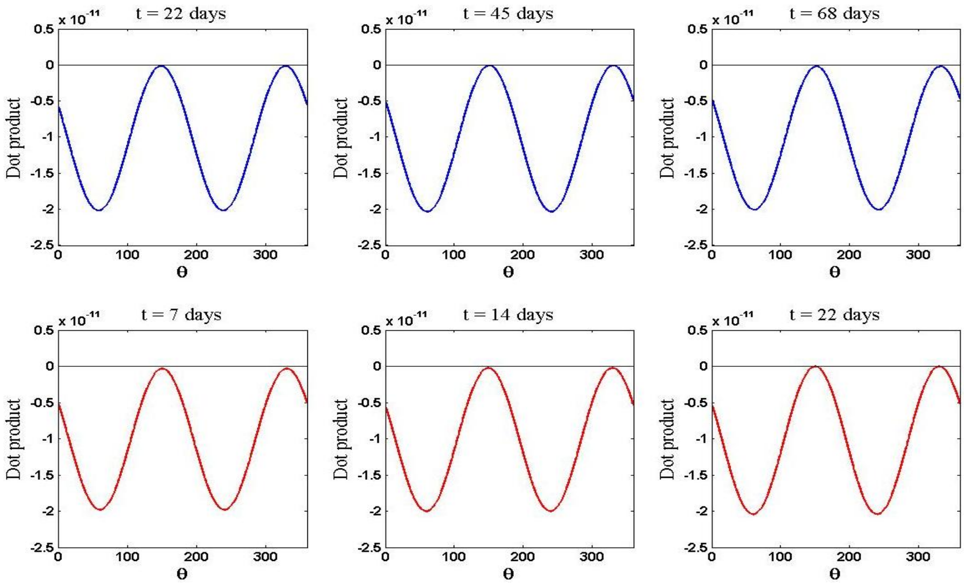

Figure 6. Scalar product between the relative acceleration $a\,(t, \theta)$ and vector $s\,(\theta)$ as a function of angle θ at three different points along the previous long (first row) and short (second row) period orbits

and positive, this means that there are no regions with ZRRA at certain points along the periodic orbits, but there exists a set of points along these periodic orbits where the radial component of the relative acceleration is equal to zero. This fact can be seen if we compute numerically the ZRRA at three different points along the previous periodic orbits (see Figure 11) considering a sphere of radius equal to 1.0 km.

The quality behaviour of the scalar product function is practically the same for all the values of t where there appear two maxima and two minima, respectively. However, there is a set of points in these trajectories where the radial component of the function $a\,(t, \theta)$ is zero with vertex at $x_h\,(t)$ for two different values of θ which we have denoted by θ^* and θ^{**}, where $\theta^* \leq \theta^{**}$ (since the scalar product function is periodic with respect to θ, the other two values of θ represent the same situation). Therefore, in principle, a set of aligned spacecrafts placed on one of them will keep fixed their mutual distances. So, Figures 12(a) and 12(b) show the function θ related to the relative position such that the radial component is maximum for all the values of t along the previous long and short period orbits, respectively. On other hand, Figure 13 shows the function θ for the values where this function has a minimum or a zero for the long (first row) and short (second row) period orbits. As we can see in Figure 13, when we do not compute the points where the scalar product function has a zero but the points where this function has a minimum, the function θ is smooth. But, when we compute the points where the scalar product function has a zero, there are two points where this function is not smooth. This fact will produce a higher cost to maintain fixed the constellation as we will show later.

4. Conclusions

The Zero Relative Radial Acceleration Lines determine the relative position, represented by an angle θ, of a satellite with respect to a nominal trajectory, where the relative acceleration of the spacecraft does not have radial component. This fact implies that once the satellite is placed at this region, the separation from the nominal trajectory will be shorter

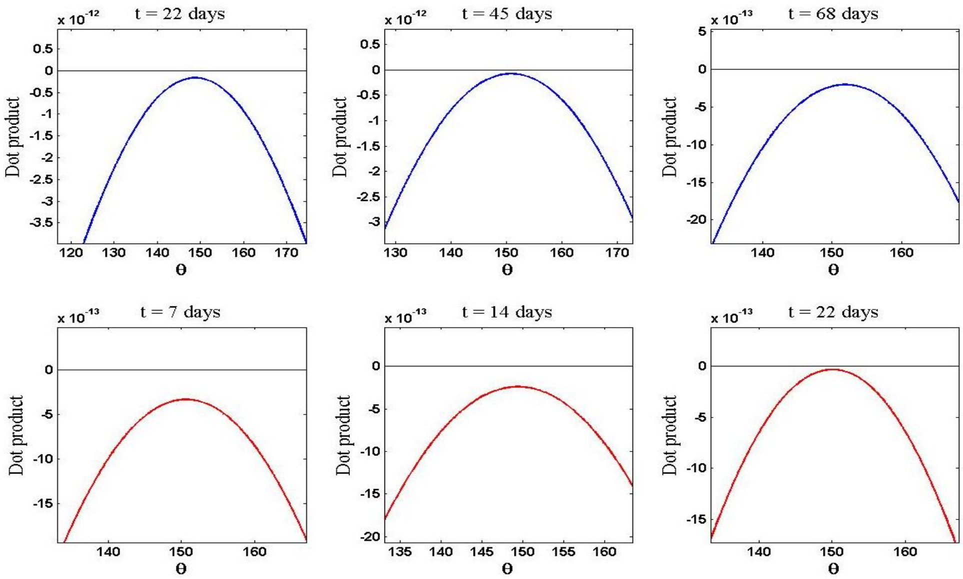

Figure 7. Scalar product between the relative acceleration $a(t, \theta)$ and vector $s(\theta)$ as a function of angle θ at three different points along the previous long (first row) and short (second row) period orbits

than than if the relative acceleration of the satellite had radial component. In principle, the cost to maintain it along the nominal trajectory will be minimum.

5. Acknowledgements

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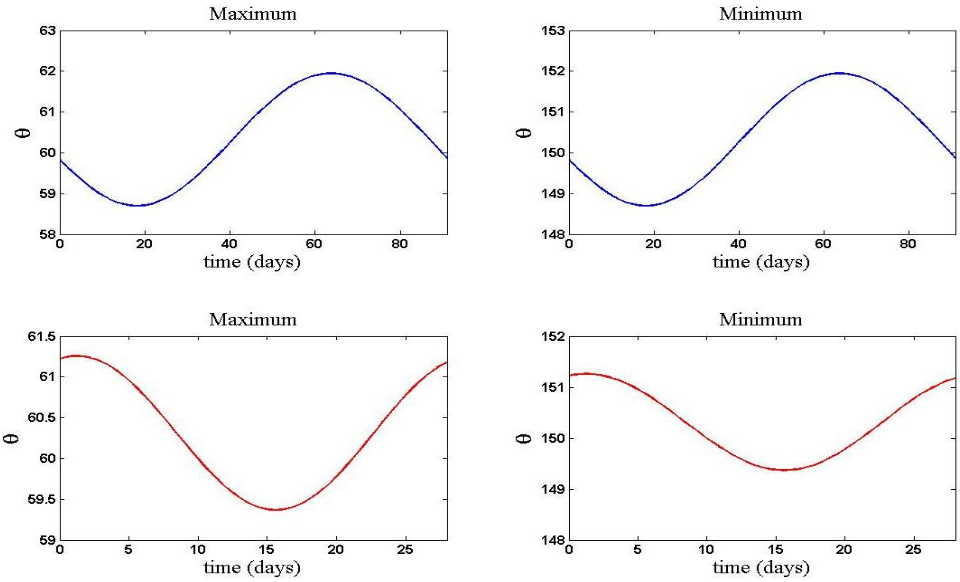

Figure 8. Maximum (left) and minimum (right) for the previous long (first row) and short (second row) period orbits

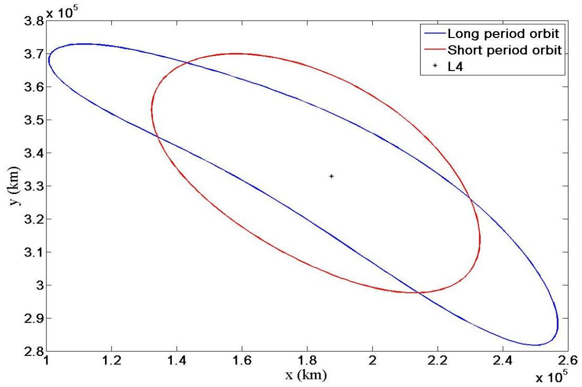

Figure 9. (a) Long and (b) short period orbits around $L4$.

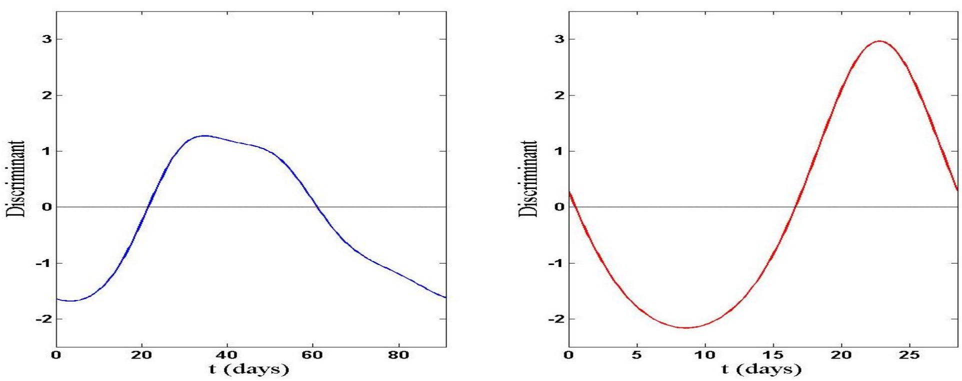

Figure 10. Discriminant of the sub-matrix F associated to the points along the previus: (a)long and (b)short period orbits

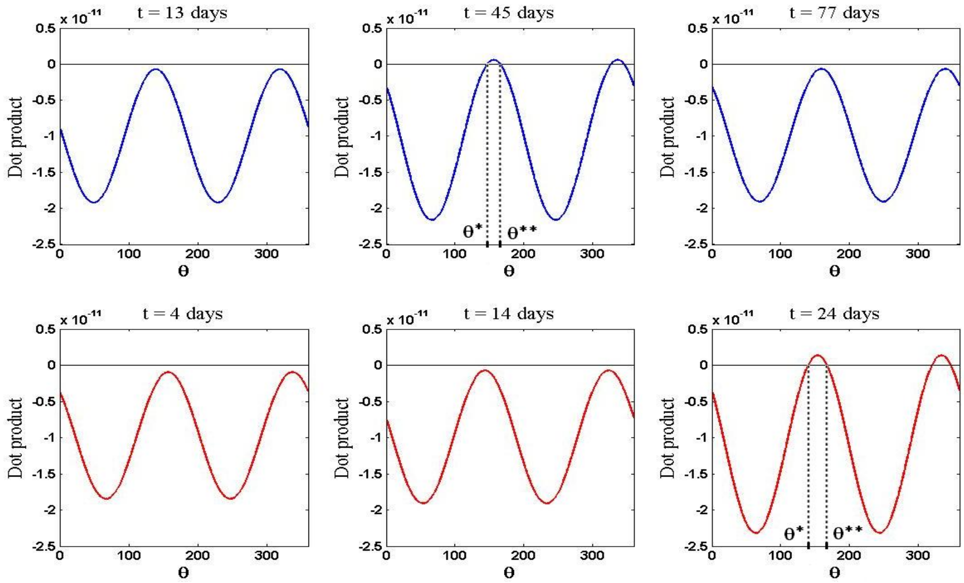

Figure 11. Scalar product between the relative acceleration $a\,(t,\theta)$ and vector $s\,(\theta)$ as a function of angle θ at three different points along the previous long (first row) and short (second row) period orbits

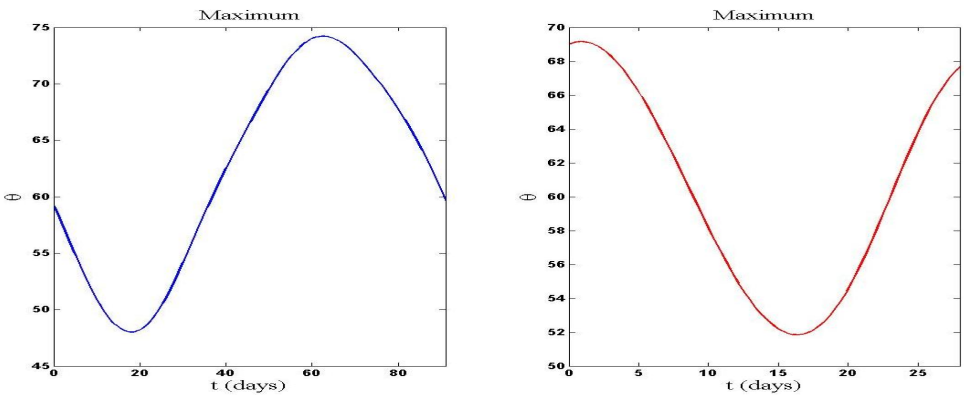

Figure 12. Maximum for the previous: (a)long and (b)short period orbits

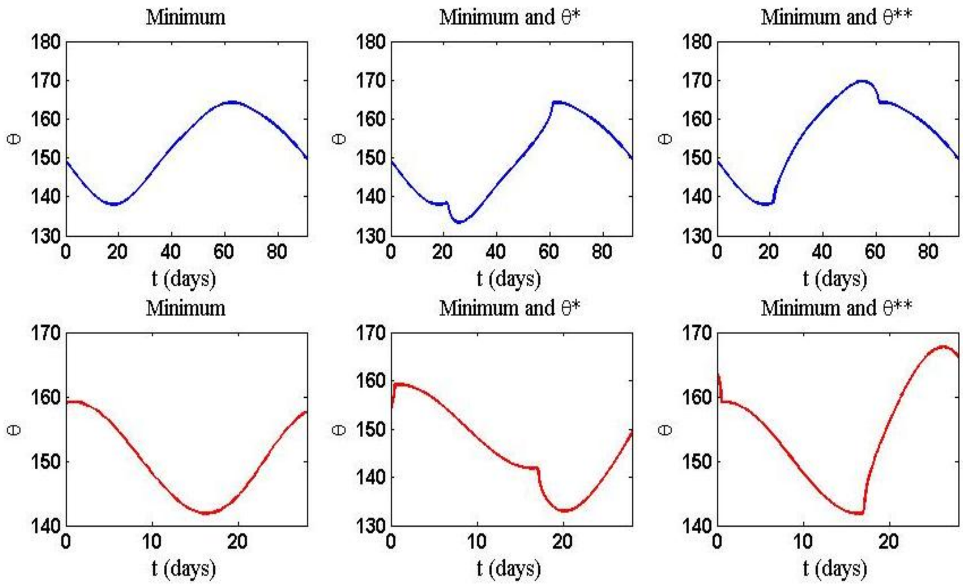

Figure 13. Minimum and ZRRA lines for the previous long (first row) and short (second row) period orbits