# Propose of a New Optimization Method of Trajectories in Chaotic Periodic Orbits


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Predicted natural processes are extreme important for diverse areas of science. Suppose, for example, the forecast of hurricane, as well, its trajectory before the formation, providing advisories, watches and warnings. The best understanding of these processes is obtained using the mathematical representations that quantify the characteristic desired. The processes changing continuously in time are called *dynamical systems*. The most common models to represent these systems are ordinary differential equations (ODE), partial differential equations (PDE), maps and automata cellular, Roth (2002).

Several systems have been observed the presence of complex behavior showing the extreme sensitivity in the initial conditions, called *chaos*, Devaney (1992). Using chaotic system it is possible move an object to any place of phase space with small perturbation (minimum energy consumed). Finding the correctly perturbation it is not easy. The solution was proposed in work Ott et al (1990) and called *chaos controlling*. Founding the sets of *unstable periodic orbits* (UPO) is the key characteristic to controlling chaos. The infinite number of UPO's embedded in a chaotic invariant set provides a skeleton of the set, and many invariants interest, such as Lyapunov exponents, fractal dimensions, etc., Davidchack and Lai (1999).

Several works of literature presents methods to determining UPO's, for example, Newton-Raphson method in Press and Teukolsky (1992), the method shows of Schmelcher and Diakonos (1998), the method shows of Davidchack and Lai (1992), approaches the literature methods presented by Roth (2002), etc. The propose this work using a set of UPO's to finding a trajectory to move a object from initial *source point* (*S*) to *target point* (*T*) with minimum energy cost in feasible time. Time is measured on the number of used points and the energy cost is directly proportional of the number of small perturbation.

An example of chaotic invariant set was showed in Figure 1. This system represents all unstable periodic orbits of tree period of Ikeda map and source and target points. The idea is move an object from *S* to *T* point using part of UPO's to mount a trajectory desire.

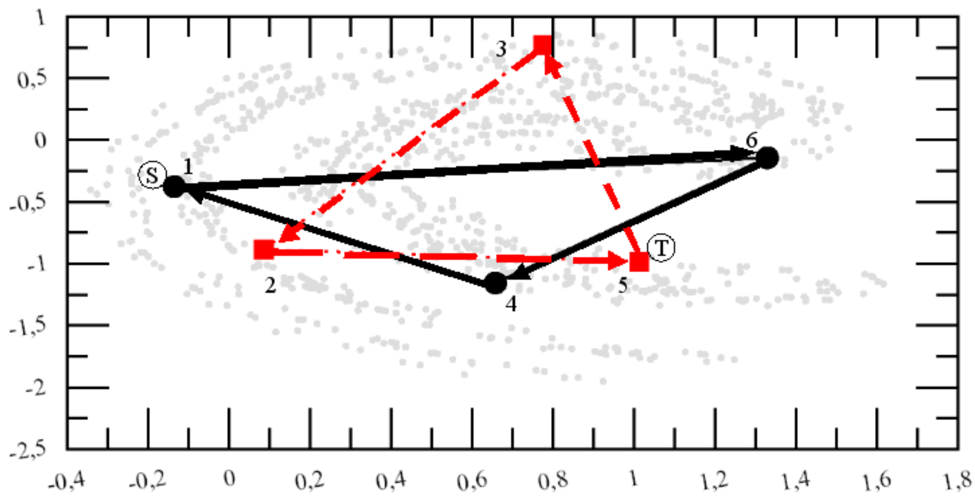


Figure 1: All unstable periodic orbits of tree period of Ikeda Map and position of S and T points

The problem, show in Figure 1, is represent by digraph show in Figure 2. The new algorithm proposed is based on branch-and-bound algorithm, presented in Lamosa (2003), to found a minimum path between *S* end *T* points for this digraph. The weight of red and blue arcs is one unity and the others arcs are determined in algorithm interaction.

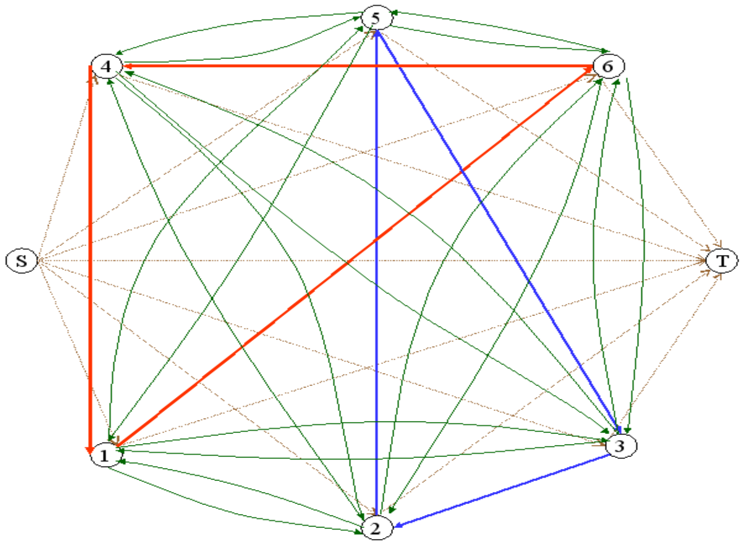


Figure2: Digraph represents all possibilities paths to Figure 1

When more orbits are included in this problem, the complexity exponentially increases. One possibility is use a high performance system to find a solution in less time.

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