

Phase Synchronization in Chaotic Systems

Rosangela Follmann¹ Elbert E. N. Macau²

¹Postgraduate Program in Applied Computing - CAP

²Laboratory for computing and Applied Mathematics-LAC

Brazilian National Institute for Space Research - INPE

C. Postal 517 - 12245970 - São José dos Campos - SP

BRAZIL

E-mail: rosangela@lac.inpe.br elbert@lac.inpe.br

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Synchronization is a basic nonlinear phenomenon of Nature, discovered at the beginning of the modern age of science by Huygens (1673). He observed that two pendulum clocks placed near each other soon become synchronized by a tiny coupling force transmitted by vibrations in the wall to which they are attached. In classic sense, synchronization means adjustment of frequencies of periodic oscillators due to a weak interaction, *i.e.*, a coupling. Synchronization in coupling chaotic systems means the appearance of some relations between phases and amplitude (Pecora et al., 1997), (Pecora & Carroll, 1990), (Rosenblum et al., 1996) between chaotic systems that interact with each other.

In this work we investigate the question about how to properly define phase in the context of a chaotic system. For a periodic system, phase is introduced as a variable defined along a limit cycle which increasing monotonically and acquire 2π at each rotation (Pikovsky et al., 2001). For a chaotic system, this concept can be extended by using three different approaches: (i) A linear function in time which at each rotation it increases by 2π ; (ii) As the angle between the projection of the phase point on the plane in a properly defined plane; (iii) As analytic signal based on the Hilbert transform. This definitions are tested to the Rössler system which have a chaotic behavior.

$$\begin{aligned}\dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= f + z(x - c).\end{aligned}\tag{1}$$

where a , f and c are parameters of the system.

We also study the influence of a periodic force acting on the system (1). This influence can be observed by constructing the synchronization region, named as Arnold tongues (Ecke et al., 1989). The Arnold tongues describe regions on the parameter space where the oscillator is locked at the phase of a periodic forcing. Another form to identify synchronization is by using a statistical approach. In this case the chaotic trajectory is observed stroboscopically in the proper phase of the external force. In the synchronous state the probability distribution of the oscillator phase is localized near some value. In the non-synchronous state the phase is spread along the attractor.

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