FUZZY MODEL OPTIMIZATION BASED ON NELDER-MEAD SIMPLEX METHOD APPLIED TO IDENTIFICATION OF A CHAOTIC SYSTEM

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Abstract. *Nelder-Mead simplex method in conjunction with Takagi-Sugeno (T-S) fuzzy model are employed to nonlinear identification of a chaotic system in this paper. Techniques such as Takagi-Sugeno fuzzy modeling have been employed in many applications due the difficulties found out during the identification task of nonlinear systems. The premise part of production rules is optimized here by using the Nelder-Mead simplex method. In turn, least mean squares technique is applied to the consequent part of a T-S fuzzy model. Experimental application using data supplied by chaotic system called Chua's circuit is analyzed. According to numerical results the Least Mean Square technique and Nelder-Mead simplex method succeeded in constructing a T-S fuzzy model for a nonlinear identification in this particular application.*

Keywords: *Nelder-Mead Simplex, Takagi-Sugeno (T-S) Fuzzy model, Least Mean Square, Optimization, Nonlinear identification.*

1. INTRODUCTION

System identification is a procedure to identify a model of an unknown process, for purposes of forecasting and/or understanding the dynamical behavior. One of the central problems in system identification is to find out a set of model structures for a certain model to approximate of a given dynamical process (Elshafei and Karray, 2005). There are three standard approaches for building mathematical models: white-box modeling (physical modeling), black-box modeling (data identification or modeling) and grey-box modeling (Ljung, 1987). A model describes reality in some way, and system identification is the theory of how mathematical models for dynamical systems are constructed from observed data. This paper is interested in black-box modeling which is designed entirely from data using no physical or verbal insight and, due to that, the resulting model lack physical or verbal significance. The structure of the model is chosen from families of models that are known to be very flexible and successful in past applications and the parameters are tuned to fit the observed data as closely as possible (Abdelazim and Malik, 2005).

The conception of mathematical models for the representation of nonlinear systems is an important procedure and has practical applications. However, the construction of mathematical models adjusted for engineering purposes is generally not a simple task. Exhaustive efforts have been devoted to developing techniques for the identification of nonlinear systems. Since the mid 1980s, Nonlinear techniques using fuzzy logic have been widely applied in many applications in system identification and, in particular, to identify complex, nonlinear systems (Abonyi et al., 2000; Montoya et al., 2001; Akkizidis and Roberts, 2001; Hadjili and Wertz, 2002). Theoretical justification of fuzzy model as a universal approximator has been given in the last decade (Kosko, 1994; Rovatti, 1998). A central feature of fuzzy modelling is that they are based on the concept of fuzzy coding of information and operating with fuzzy sets instead of numbers. Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno, 1985; Sugeno and Kang, 1988), for instance, exhibits both high nonlinearity and a simple structure. As reported in the literature, it is capable of approximating a complex system using fewer fuzzy rules than conventional Mamdani-type fuzzy models.

The identification problem in T-S modeling consists of two major parts: structure identification and parameter identification. The structure identification is related to both the determination of the premise part and the consequent part of the production rules. It consists of determining the premise space partition and extracting the number of rules and determining the structure of the output elements (equations), respectively. Finally, the parameter-learning task consists of determining the system parameters, i.e., membership functions, so that a performance measure based on the output errors is minimized.

Over the past few years an increasing number of optimization methodologies have been employed in tuning and design of fuzzy models, such as genetic algorithms (Linkens and Nyongesa, 1996; Bonissone, 1999), descent gradient-based method (Cerrada et al., 2005), and particle swarm optimization (Marinke et al., 2005; Araujo and Coelho, 2006). In this context, this paper combines fuzzy T-S model and Nelder-Mead simplex method (Nelder and Mead, 1965) in such a way that a fuzzy T-S model is tuned by a Nelder-Mead Optimization (NMO) approach.

The objective of this paper is to generate an optimized fuzzy model in order to describe the dynamical behavior of a Chua's oscillator and so to explore the effectiveness of NMO approach in constructing T-S fuzzy models for nonlinear identification. Chua's oscillator consists of a simple electronic circuit that is capable of producing chaos, as well as can exhibit a vast array of behaviors including an assortment of steady-states, bifurcations and routes to chaos (Forbes et al., 2005). The Chua's oscillators have been widely used as platform of tests in many areas related to the study of chaos, in chaotic secure communication systems, chaotic spread-spectrum communications, and some other fields.

The structure identification of the premise and the consequent part of production rules of T-S fuzzy system are carried out independently by distinct methods in this paper. The T-S fuzzy system design employs Nelder-Mead's simplex algorithm for figuring out the premise part meanwhile least mean squares is used for the calculus of consequent part of production rules of a T-S fuzzy system for nonlinear identification.

2. FUNDAMENTALS OF TAKAGI-SUGENO FUZZY SYSTEM

Fuzzy models focus on the use of heuristics in the system description. They can be seen as logical models that use "IF-THEN" rules to establish qualitative relationships among variables. Their rule-based nature allows the use of information expressed in the form of natural language statements (Vernieuwe et al., 2005).

In this context, T-S models have recently become a powerful practical engineering tool for modeling and control of complex systems. The T-S model representation often provides efficient and computationally attractive solutions to a wide range of modeling problems introducing a powerful multiple model structure that is capable to approximate nonlinear dynamics, multiple operating modes and significant parameter and structure variations.

The essential idea of T-S fuzzy model is the partitioning of the input space into fuzzy areas and the approximation of each area through a linear model in such a way that a global nonlinear model is computed. It is characterized as a set of IF-THEN rules where the consequent part are linear sub-models describing the dynamical behaviour of distinct operational conditions meanwhile the antecedent part is in charge of interpolating these sub-systems. The "IF statements" define the premise part that is featured as linguistic terms while the THEN functions constitute the consequent part of the fuzzy system characterized, but not limited, as linear polynomial terms. The global model is then obtained by the interpolation between these various local models. This model can be used to approximate a highly nonlinear function through simple structure using a small number of rules. The T-S models consist of linguistic IF-THEN rules that can be represented by the following general form:

$$R^{(j)} : \text{IF } z_i \text{ IS } A_1^j \text{ AND} \ldots \text{AND } z_m \text{ IS } A_m^j \text{ THEN } g_j = w_0^j + w1^j u_1^j + \ldots + w_{qj}^j u_{qj}^j \,. \tag{1}$$

The IF statements define the premise part while the THEN functions constitute the consequent part of the fuzzy system; $\underline{z} = [z_1, \ldots, z_m]^T$, such as $i = 1, \ldots, m$, is the input vector of the premise p, and A_i^j are labels of fuzzy sets. The parameters $\underline{u} = [u_1^j, \ldots, u_{qj}^j]^T$ represents the input vector to the consequent part of $R^{(j)}$ that comprising q_j terms; $g_j = g_j(u^j)$ denotes the j-th rule output which is a linear polynomial of the consequent input terms u_i^j , and $\underline{w} = [w_1^j, \ldots, w_{qj}^j]^T$ are the polynomial coefficients that form the consequent parameter set. Each linguistic label A_i^j is associated with a Gaussian membership function, $\mu_{A_i^j}(z_i)$, described by eq. (2) where m_{ij} and σ_{ij} are the mean value and the standard deviations of the Gaussian membership function, respectively, and represent the centers/core and the spreads/support of membership functions:

$$\mu_{A_i^j}(z_i) = \exp\left[-\frac{1}{2}\frac{(z_i - m_{ij})^2}{\sigma_{ij}^2}\right] \,. \tag{2}$$

The union of all these parameters formulates the set of premise parameters. The firing strength of rule $R^{(j)}$ represents its excitation level and, for instance, it can be chosen as:

$$\mu_j(\underline{z}) = \mu_{A_1^j}(z_1)\mu_{A_2^j}(z_2)\ldots\mu_{A_m^j}(z_m) \,. \tag{3}$$

The fuzzy sets pertaining to a rule form a fuzzy region (cluster) within the premise space, $A_i^j \times \ldots \times A_m^j$, with a membership distribution described in eq. (3). Given the input vectors z and u^j, such as $j = 1, \ldots, M$, the final output of the fuzzy system is inferred by taking the weighted average of the local outputs $g_j\left(\underline{u}^j\right)$ that is given by

$$y = \sum_{j=1}^{M} v_j(\underline{z}) g_j(\underline{u}^j) \ , \tag{4}$$

where M denotes the number of rules and $v_j(z)$ is the normalized firing strength of $R^(j)$, defined as:

$$v_j(\underline{z}) = \frac{\mu_j(\underline{z})}{\sum_{j=1}^{M} \mu_j(\underline{z})} \ . \tag{5}$$

The structure identification of T-S system is computed based on NMO for premise part optimization while the consequent part optimization is determined by batch least mean squares method (pseudo-inversion method).

2.1 Nelder-Mead Optimization for T-S Fuzzy Modeling

The simplex search method, first proposed by Spendley, Hext, and Himsworth (1962) and later refined by Nelder and Mead (1965), is a derivative-free line search method that was particularly designed for traditional unconstrained minimization scenarios, such as the problems of nonlinear least squares, nonlinear simultaneous equations, and other types of function minimization (see, e.g., Olsson & Nelson (1975)). Consider first that function values at the (n + 1) vertices of an initial simplex are evaluated, which is a polyhedron in the factor space of n input variables. In the minimization case, the vertex with the highest function value is replaced by a newly reflected, better point, which would be approximately located in the negative gradient direction (Fan et al., 2006).

Through a sequence of elementary geometric transformations (reflection, inside contraction, expansion and outside contraction), the initial simplex moves, expands or contracts. To select the appropriate transformation, the NMO method only uses the values of the function to be optimized at the vertices of the simplex considered. After each transformation, the current worst vertex is replaced by a better one (Chelouah and Siarry, 2003). Nelder-Mead simplex method procedure is illustrated in Figure 1.

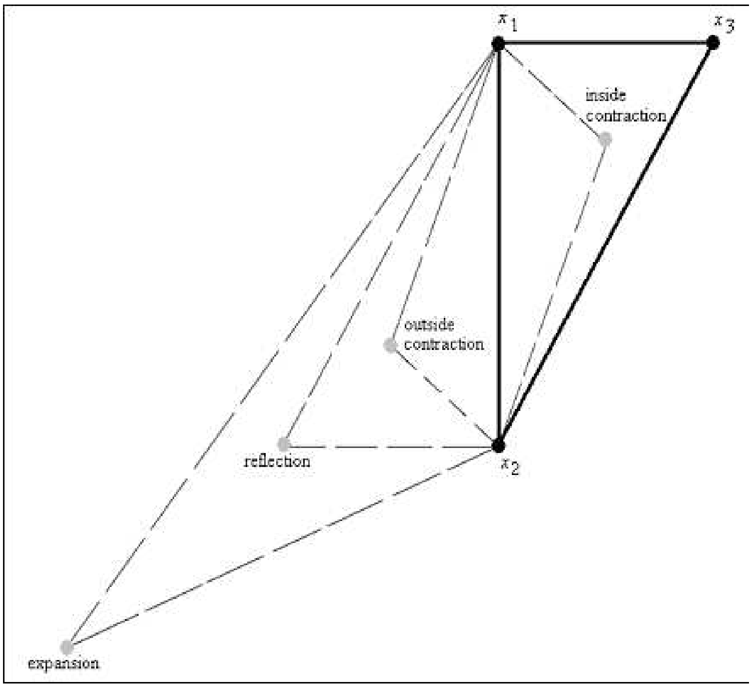


Figure 1. Nelder-Mead simplex method and the representation of new points in search space.

At the beginning of the algorithm, one moves only the point of the simplex, where the objective function is worst (this point is called “high”), and one generates another point image of the worst point. This operation is the reflection. If the reflected point is better than all other points, the method expands the simplex in this direction, otherwise, if it is at least better than the worst, the algorithm performs again the reflection with the new worst point. The contraction step is performed when the worst point is at least as good as the reflected point, in such a way that the simplex adapts itself to the function landscape and finally surrounds the optimum. If the worst point is better than the contracted point, the multi-contraction is performed. At each step we check that the generated point is not outside the allowed reduced solution space (Chelouah and Siarry, 2003). In this work, the function *fminsearch* of Matlab was employed for validation of Nelder Mead simplex method.

3. NONLINEAR IDENTIFICATION OF CHUA'S OSCILLATOR

3.1 Chua's Oscillator

In recent years, chaos control, synchronization and identification of chaotic systems, especially the Chua's oscillator, have received increasing attention from various scientific and engineering communities due to its great potential in technological applications (Tôrres and Aguirre, 1999, 2000; Thamilmaran et al., 2000; Palacios, 2002). In a simple case, the Chua circuit can be described by ordinary differential equations of the following form:

$$C_1 \frac{dv_{C_1}}{dt} = \frac{v_{C_2} - v_{C_1}}{R_{11}} - i_d(v_{C_1}) \tag{6}$$

$$C_2 \frac{dv_{C_2}}{dt} = \frac{v_{C_1} - v_{C_2}}{R_{11}} + i_L \tag{7}$$

$$L \frac{di_L}{dt} = -v_{C_2} \tag{8}$$

where R_{11} is a linear resistor, L is an inductor, v_{C_j} is the voltage on capacitor $C_j, j = 1, 2$; i_L is the current through the inductor. The static nonlinearity of Chua's diode is the piecewise linear curve given by current that passes for the Chua's diode as described in (Tôrres and Aguirre, 1999, 2000):

$$i_d(v_{C_1}) = m_0 v_{C_1} + 0.5(m_0 - m_1)\{|v_{C_1} + B_p| - |v_{C_1} - B_p|\} \tag{9}$$

where m_0, m_1 and B_p are the parameters. The nonlinear characteristic can be realized using operational amplifiers (Tôrres and Aguirre, 2000), as shown in Figure 2.

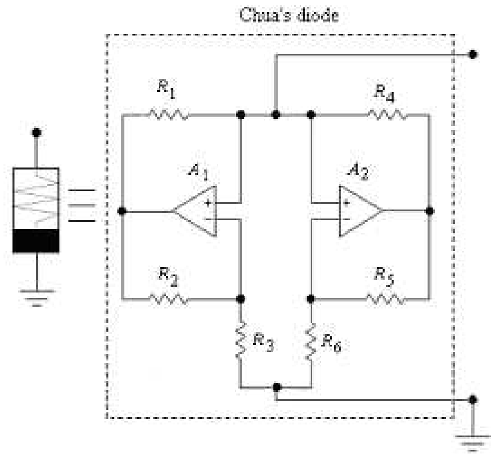


Figure 2. Representation of Chua's diode (Tôrres and Aguirre, 2000).

The construction of this circuit can be realized by the composition of a network of linear passive elements connected to a nonlinear active component called Chua's diode, as illustrated in the circuit of the Figure 3. The conception of this inductorless Chua's circuit structure was based in Tôrres and Aguirre (1999, 2000). It is worth mentioning that different values for the components of the electric circuit result in attractors with different geometry.

3.2 Identification of T-S Fuzzy Model

Identification of dynamic systems can be performed with a series-parallel or parallel model. Series-parallel structure is the type of mathematical model adopted for identification (one-step ahead forecasting) of Chua's oscillator when using the hybrid NMO-TS modelling approach as shown in Figure 4.

The estimated T-S fuzzy model output based on NMO, $\hat{y}(k)$, used for computing the minimum square error when compared with the actual output, $y(k)$ was computed by using one-step ahead forecasting. Denote n_y, n_u, and n_n as the time maximum lags of the model output, control input, and noise, respectively. Depending on the time-lagged inputs that

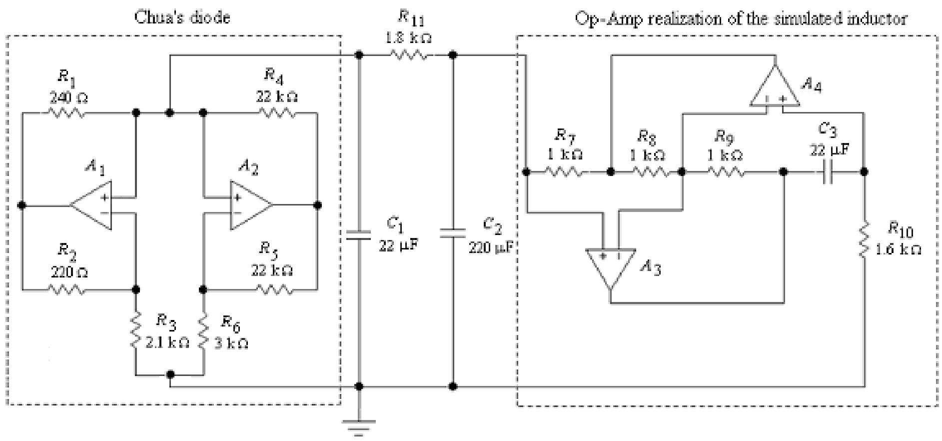


Figure 3. Implementation of Chua's oscillator.

are used for the T-S fuzzy model, different configurations of models can be used. In this work, a NAR (Nonlinear Auto Regressive) model was adopted, given by:

$$\hat{y}(k) = f_{TS}(u(k-1), \ldots, u(k-n_u), y(k-1), \ldots, y(k-n_y), \theta) \tag{10}$$

where the unknown nonlinear function f_{TS} is the TS fuzzy model of the system, k represents the kth instant of time, and θ is the estimated vector of parameters for the model.

One of the most important tasks in building an efficient forecasting model based in T-S fuzzy model is the selection of the relevant input variables. The input selection problem can be stated as follows: among a large set of potential input candidates, choose those variables that highly affect the model output. Unfortunately, there is no systematic procedure currently available which can be followed in all circumstances. In this work, input selection is heuristically performed. The inputs of T-S fuzzy system are process output and control input signals of reduced order with $n_y = 2$, $n_u = 0$, and $n_n = 0$. Aditionally, the input vectors for the T-S fuzzy system are $[y(k-2); y(k-1)]$ while the model output is $\hat{y}(k)$.

The Chua's oscillator experimental data (voltage on capacitor C1) shown in Figure 5 was employed to elicit the fuzzy model through Nelder-Mead approach. The first part of 1000 samples were used during training (estimation) phase of T-S system design while the other 1160 in validation (test or generalization) phase.

Although, NMO allows to extract the number of rules and to determine the premise and consequent elements, here this method is applied to obtain membership functions and thus to determine the premise space partition.

Setting up this parameter as 2 or 3 production rules, NMO needs to deal with a vector of decision variables whose elements are 9 or 12 centres and 2 or 3 spreads of a Gaussian function, respectively, core and support of membership functions. In this case, the spread of Gaussian membership function adopted for each input of vectors $[y(k-2); y(k-1)]$ of T-S fuzzy model is the same.

The system identification by T-S fuzzy model is appropriate if a suitable performance index is available according to

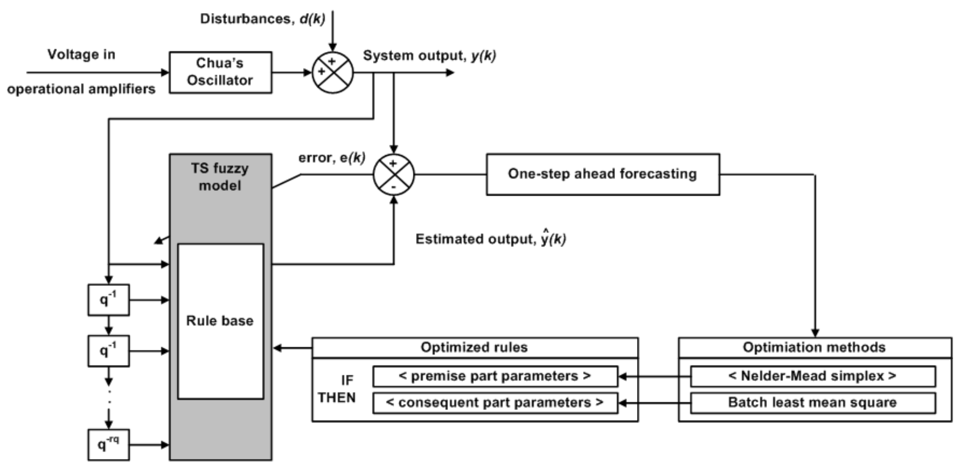


Figure 4. One-step ahead forecasting using T-S fuzzy model using NMO and batch mean least squares.

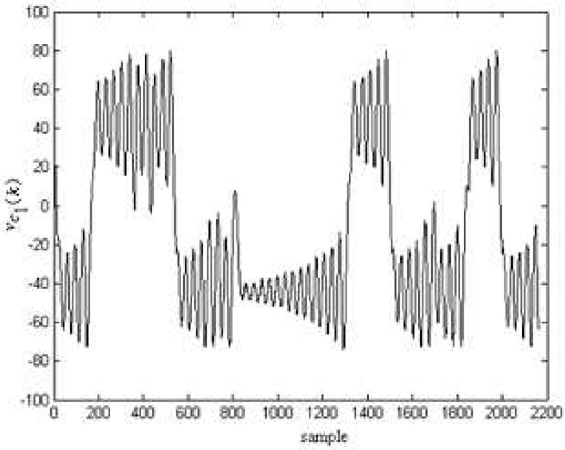


Figure 5. Experimental data of Chua's oscillator for identification task.

the necessities of users. The performance criterion chosen for evaluate the relationship between the real output and the estimate output during the optimization process (maximization problem) was the Pearson multiple correlation coefficient index. This coefficient represents the R^2 of training phase of T-S fuzzy model computed as given by:

$$R^2_{training} = 1 - \frac{\sum_{k=1}^{0.5Ne} \left[y(k) - \widehat{y}(k)\right]^2}{\sum_{k=1}^{0.5Ne} \left[y(k) - \overline{y}\right]^2} \tag{11}$$

where N_e is the total number of samples evaluated in estimation phase, and $\overline{y}$ is the system real output. When $R(.)^2$ is close to unit, $R(.)^2 = 1.0$, a sufficient accurate model for the measured data of the system is found. A $R(.)^2$ between 0.9 and 1.0 is suitable for applications in identification and model-based control.

In this context, the performance evaluation of validation phase of optimized T-S fuzzy system is realized by:

$$R^2_{validation} = 1 - \frac{\sum_{k=0.5Na+1}^{Ne} \left[y(k) - \widehat{y}(k)\right]^2}{\sum_{k=0.5Ne+1}^{Ne} \left[y(k) - \overline{y}\right]^2} \ . \tag{12}$$

where N_v is the total number of samples evaluated in validation phase. Based on values of R^2 in training and validation phases, the NMO uses an objective function given by maximization of harmonic mean of these R^2 values and it is called R^2_{hm}.

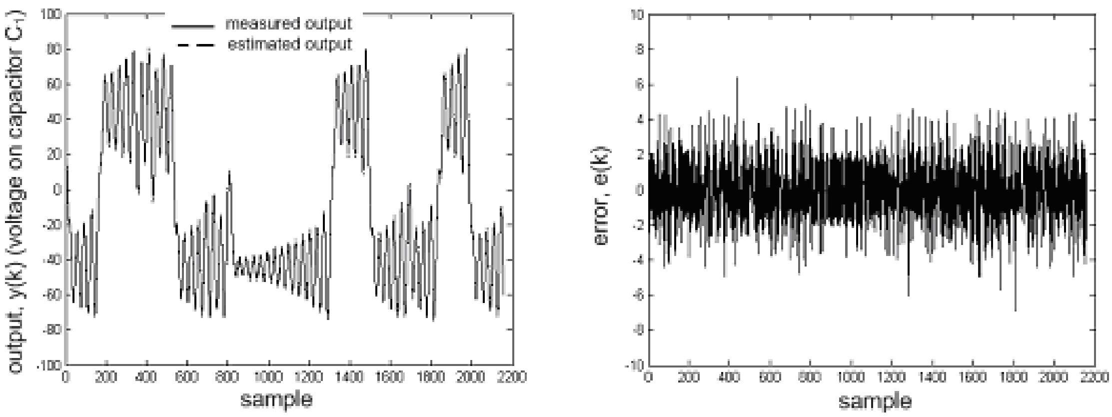
The NMO used in the computational simulations was the *fminsearch* function of Matlab (MathWorks). To illustrate the effectiveness of the T-S fuzzy model several simulations were performed. The programs were run on a 3.8 GHz Pentium IV processor with 2 GB of RAM. In each case study, 30 independent runs were made with the NMO based on 30 different initial trial solutions.

The results (best of 30 independent runs with 100 evaluations of objective functions in each run) for the NMO of T-S fuzzy system design in training and validation phases are presented in Table table:table1.

The best performance index R^2_{hm} was obtained by T-S with 2 production rules. The results using T-S with 2 production rules present small standard deviation and also the best $R^2_{training}$ and $R^2_{validation}$ mean value (best convergence property) as it is presented in Table 1.

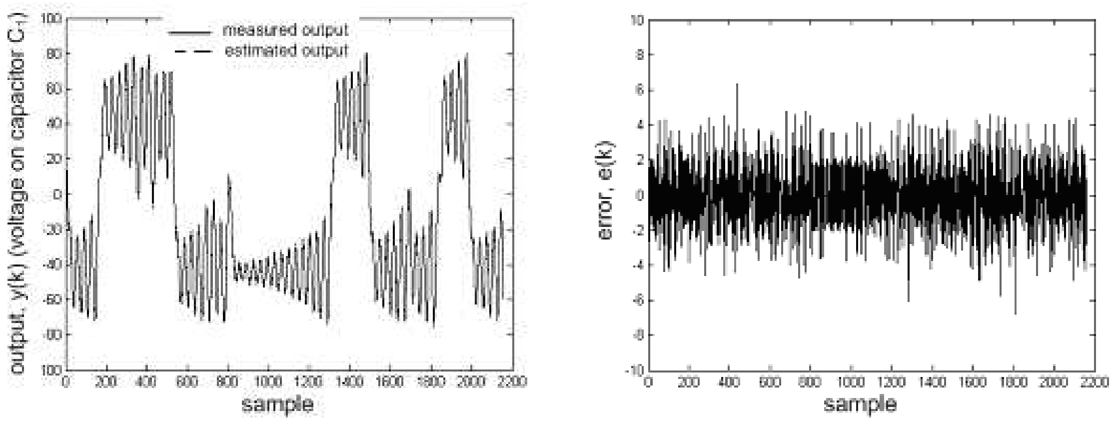
Table 1. Results (best of 30 independent runs) for the NMO of T-S fuzzy system design in training phase.

Production Rules	Performance Index	Minimum (worst)	Maximum (best)	Mean	Median	Standard Desviation
2	$R^2_{training}$	0.8246	0.9984	0.9748	0.9983	0.0577
2	$R^2_{validation}$	0.8221	0.9981	0.9739	0.9980	0.0584
2	R^2_{hm}	0.8234	0.9984	0.9741	0.9981	0.0581
3	$R^2_{training}$	0.0079	0.9984	0.9368	0.9970	0.2088
3	$R^2_{validation}$	0	0.9982	0.9356	0.9966	0.2109
3	R^2_{hm}	4.4×10^{-16}	0.9983	0.9361	0.9968	0.2106



(a) Dynamical Response. (b) Error.

Figure 6. Best response obtained through resulting T-S model with 2 production rules based on NMO.



(a) Dynamical Response. (b) Error.

Figure 7. Best response obtained through resulting T-S model with 3 production rules based on NMO.

NMO-TS fuzzy models with 2 and 3 production rules achieved a good approximation for experimental data in training and validation phases. Continuous and dashed lines represent measured and simulated outputs in results presented in Figures 6 and 7. Experimental results had shown that the integrating NMO and T-S fuzzy system presented successful results due precision in one-step ahead predicting nonlinear dynamics.

4. CONCLUDING REMARKS AND FUTURE RESEARCH

Fuzzy models represent attractive platforms to model nonlinear systems since they can work in sinergy with distinct efficient tuning algorithms. In this work, a NMO approach is evaluated in T-S fuzzy system modeling and design.

The elicited fuzzy model with only two membership functions determining the premise space partition demonstrated its effectiveness in emulating the time response for the Chua's oscillator. The computational performance and the accuracy of the proposed methodology integrating T-S fuzzy model and NMO indicates that the proposed approach is suited for applications in the design of nonlinear identification models for a wide class of the complex systems.

Further research intends to compare the use of other learning or optmization techniques such as simulated annealing, evolutionary algorithms, artificial neural network in order to verify the improvements that can be carried out by combining the T-S fuzzy model with other tools when tuning the design parameters.

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