Recommendation for Space Data System Standards

IMAGE DATA COMPRESSION

RECOMMENDED STANDARD

CCSDS 122.0-B-1

BLUE BOOK
November 2005
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1 INTRODUCTION

1.1 PURPOSE

The purpose of this document is to establish a Recommended Standard for a data compression algorithm applied to two-dimensional digital spatial image data from payload instruments and to specify how this compressed data shall be formatted into segments to enable decompression at the receiving end.

Source coding for data compression is a method utilized in data systems to reduce the volume of digital data to achieve benefits in areas including, but not limited to,

a) reduction of transmission channel bandwidth;

b) reduction of the buffering and storage requirement;

c) reduction of data-transmission time at a given rate.

1.2 SCOPE

The characteristics of instrument data are specified only to the extent necessary to ensure multi-mission support capabilities. The specification does not attempt to quantify the relative bandwidth reduction, the merits of each approach discussed, or the design requirements for coders and associated decoders. Some performance information is included in reference [B1].

This Recommended Standard addresses image data compression, which is applicable to a wide range of space-borne digital data, where the requirement is for a scalable data reduction, including the option to use lossy compression, which allows some loss of fidelity in the process of data compression and decompression. Reference [B1] gives an outline for an implementation.

1.3 APPLICABILITY

This Recommended Standard applies to data compression applications of space missions anticipating packetized telemetry cross support. In addition, it serves as a guideline for the development of compatible CCSDS Agency standards in this field, based on good engineering practice.

1.4 RATIONALE

The concept and rationale for the Image Data Compression algorithm described herein may be found in reference [B1].
1.5 CONVENTION

In the specification of headers for compressed data, the following convention is used to identify each bit in an $N$-bit word. The first bit in the word to be transmitted (i.e., the most left justified when drawing a figure) is defined to be ‘Bit 0’; the following bit is defined to be ‘Bit 1’ and so on up to ‘Bit $N-1$’. When the word is used to express an unsigned binary value (such as a counter), the Most Significant Bit (MSB) shall correspond to the highest power of two, i.e., $2^{N-1}$.

NOTE – This bit numbering convention applies to segment headers described in 4.2. A different numbering convention is used to index magnitude bits in Discrete Wavelet Transform (DWT) coefficients, as described in 4.1.

![Bit numbering convention diagram]

In accordance with modern data communications practice, spacecraft data words are often grouped into eight-bit ‘words’ that conform to the above convention. Throughout this Recommended Standard, the following nomenclature is used to describe this grouping:

8-Bit Word = ‘Byte’

1.6 DEFINITIONS

In this document, for any real number $x$, the largest integer $n$ such that $n \leq x$ shall be denoted by

$$n = \lfloor x \rfloor,$$

and correspondingly, the smallest integer $n$ such that $n \geq x$ by

$$n = \lceil x \rceil.$$

The modulus $m$ of a number $M$ with respect to a divisor $n$ is denoted by

$$m = M \mod n.$$

The statement that a value $M$ is coded mod $n$ means the number

$$m = M \mod n$$

is coded instead of $M$. 
1.7 NOMENCLATURE

The following conventions apply throughout this Recommended Standard:

a) the words ‘shall’ and ‘must’ imply a binding and verifiable specification;

b) the word ‘should’ implies an optional, but desirable, specification;

c) the word ‘may’ implies an optional specification;

d) the words ‘is’, ‘are’, and ‘will’ imply statements of fact.

In the normative sections of this document (sections 3 and 4), informative (i.e., non-normative) text is differentiated from normative text through the following convention:

a) notes introduce brief informative statements;

b) the following headings introduce one or more paragraphs of informative text:
   – Overview;
   – Rationale.

1.8 REFERENCES

This Recommended Standard contains no normative references. Informative references are listed in annex B.
2 OVERVIEW

2.1 GENERAL

This Recommended Standard defines a particular payload image data compression algorithm that has widespread applicability to many types of instruments. This Recommended Standard does not attempt to explain the theory underlying the operation of the algorithm, which is partially addressed in [B1].

There are two classes of data compression methods: lossless and lossy. Under lossless compression, the original data can be reproduced exactly, while under lossy compression, quantization or other approximations used in the compression process result in the inability to reproduce the original data set without some distortion. The increased information content of data subjected to lossless compression results in a larger volume of compressed data for a given source data set.

The compression technique described in this Recommended Standard can be used to produce both lossy and lossless compression. An alternative lossless compression technique, which has lower complexity but is not specifically tailored for imagery, has been previously adopted by CCSDS (see reference [B2]). The JPEG2000 standard is another image compressor (see reference [B6]). The present Recommended Standard differs from the JPEG2000 standard in several respects:

a) it specifically targets high-rate instruments used on board of spacecraft;

b) a trade-off has been performed between compression performance and complexity with particular emphasis on spacecraft applications (cf. a);

c) the lower complexity of this recommendation supports fast and low-power hardware implementation;

d) it has a limited set of options, supporting its successful application without in-depth algorithm knowledge.

The compressor consists of two functional parts, depicted in figure 2-1, a Discrete Wavelet Transform module that performs decorrelation, described in section 3, and a Bit-Plane Encoder which encodes the decorrelated data, described in section 4.

![Figure 2-1: General Schematic of the Coder](image-url)
This Recommended Standard supports both frame-based input formats produced, for example, by CCD arrays (called image frames) and strip-based input formats produced by push-broom type sensors (called image stripmaps). An image pixel dynamic range of up to 16 bits is supported. The algorithm specified supports a memory-effective implementation of the compression procedure which does not require large intermediate frames for buffering (see reference [B1]).

2.2 DATA DELIVERY

The encoded bitstream corresponding to an image frame or stripmap, consists of a single segment or a sequence of segments. Each segment consists of a header followed by a coded data field. A segment can have either fixed length or variable length depending on the operational mode selected. Values of algorithm parameters used in encoding can be specified using the header defined in section 4.

The effects of a single bit error can propagate to corrupt reconstructed data to the end of the affected segment (see reference [B1]). Therefore measures should be taken to minimize the number of potential bit errors on the transmission link. The transport mechanism for the delivery of the encoded bitstream shall support, in the event of a bit error, the ability to relocate the header of the next segment.

In case the encoded bitstream is to be transmitted over a CCSDS space link, several protocols can be used to transfer the sequence of segments. Those protocols are specified in references:

- reference [B3], Space Packet Protocol;
- reference [B4], CCSDS File Delivery Protocol (CFDP);
- reference [B5], packet service or bitstream service as provided by the AOS Space Data Link Protocol.

Interface to those space link protocols and related issues are discussed in reference [B1].
3 DESCRIPTION OF THE DISCRETE WAVELET TRANSFORM

3.1 OVERVIEW

This Recommended Standard for the decorrelation module makes use of a three-level, two-dimensional (2-d), separable Discrete Wavelet Transform (DWT) with nine and seven taps for low- and high-pass filters, respectively. Such a transform is produced by repeated application of a one-dimensional (1-d) DWT described in 3.3, 3.5, and 3.7. Two specific 1-d wavelets are specified with this Recommended Standard: the 9/7 biorthogonal DWT, referred to as ‘9/7 Float DWT’ or simply ‘Float DWT’, and a non-linear, integer approximation to this transform, referred to as ‘9/7 Integer DWT’ or simply ‘Integer DWT’.

The definition of the functional inverse of either DWT is included in 3.4, 3.6, and 3.8.

While the Float DWT generally exhibits superior compression efficiency in the lossy domain, only the Integer DWT supports strictly lossless compression.

Image data is assumed to use $R$-bit pixels to represent either signed or unsigned integer values, where $R \leq 16$.

The values output from the 3-level 2-d DWT (see 3.7) are converted to appropriate integer values before applying the Bit Plane Encoder (BPE—see section 4). Each integer is represented using a binary word consisting of a single sign bit along with several magnitude bits. The maximum word size necessary to store each such integer depends on $R$.

In the case of the Float DWT, the computed wavelet domain values are rounded to the respective nearest integers before applying the BPE. A corresponding word length of $R+5$ bits is adequate to store these integer values.

In the case of the Integer DWT, before applying the BPE, the computed wavelet domain values are multiplied by integer weights that are uniform in each subband (see 3.9). A corresponding word length of $R+4$ bits is adequate to store such integers before the weighting factors are applied. Following the weighting operation, longer words might be used to store wavelet coefficients in some implementations.

Implementation schemes and image reconstruction methods are not part of this Recommended Standard, although some approaches are outlined in reference [B1]. Compressor implementations can differ considerably depending, for example, on whether the implementation waits for an entire frame to be formed before beginning compression.
3.2 IMAGE FRAME

3.2.1 Calculation of both the Float DWT and Integer DWT depends on the dimensions, i.e., row width and column height, of the image frame being transformed.

NOTE – This formal consideration of frame size does not imply that an implementation has to provide buffers for complete frames.

3.2.2 Image width shall be communicated to the decoder using the segment headers defined in 4.2. The image height is inferred indirectly via a header flag indicating the last segment of an image, as described in 4.2.

NOTE – Although the compressor will operate on small images, it is designed to exploit two-dimensional correlations in the image, and consequently, compression performance may be poor when either image height or width is smaller than 32.

3.2.3 The maximum image width (number of columns) is $2^{20}$; the minimum image width is 17. There is no restriction on the maximum number of rows that constitute one image frame; the minimum number of rows is 17.

3.2.4 The application of either Float or Integer DWT to an image requires that the image dimensions be integer multiples of eight. If the width or height of the original image is not a multiple of eight, the image shall be ‘padded’ by appending the minimum number of extra rows and/or columns to produce an image with both width and height divisible by eight:

a) when columns are added for padding,

1) columns shall be appended at the right edge of the image, i.e., to the edge having the highest pixel index in the horizontal direction,

2) pixel values of appended columns shall be copies of the value of the right-most image column;

b) when rows are added for padding,

1) rows shall be appended at the bottom edge of the image, i.e., to the edge having the highest pixel index in the vertical direction,

2) appended rows shall be identical copies of the last row to the image.

3.2.5 Rows and/or columns of data added for padding shall be discarded after decompression.
3.3 ONE-DIMENSIONAL SINGLE-LEVEL DWT

3.3.1 9/7 FLOAT TRANSFORM

3.3.1.1 Analysis Filter Coefficients

The 9/7 Float DWT uses two sets of analysis filter coefficients (‘taps’):

\[
\{h_{-4}, h_{-3}, h_{-2}, h_{-1}, h_0, h_1, h_2, h_3, h_4\}
\]

\[
\{g_{-3}, g_{-2}, g_{-1}, g_0, g_1, g_2, g_3\}
\]

The numerical values of the analysis filter coefficients are specified in table 3-1.

Table 3-1: Analysis Filter Coefficients for the 9/7 Filter

<table>
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<tr>
<td>0</td>
<td>0.852698679009</td>
<td>-0.788485616406</td>
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<td>±1</td>
<td>0.377402855613</td>
<td>0.418092273222</td>
</tr>
<tr>
<td>±2</td>
<td>-0.110624404418</td>
<td>0.040689417609</td>
</tr>
<tr>
<td>±3</td>
<td>-0.023849465020</td>
<td>-0.064538882629</td>
</tr>
<tr>
<td>±4</td>
<td>0.037828455507</td>
<td></td>
</tr>
</tbody>
</table>

NOTES

1. The filter coefficients are derived as approximations to real, irrational numbers. The arithmetic precision used in calculating the DWT is a matter of implementation and is not part of this Recommended Standard. The impact of reduced arithmetic precision of the filter coefficients on compression performance is mitigated by the fact that the wavelet coefficients are rounded to the nearest integer before coding. See reference [B1].

2. The 1-d DWT is defined for an even number of input samples. The image extension procedure described in 3.2 ensures that the number of samples to which the 1-d DWT is applied is even at all levels.

3.3.1.2 Analysis Filter Operations

For \(N>2\), let

\[
\{x_0, x_1, x_2, \ldots, x_{2N-1}\}
\]

denote a one-dimensional signal consisting of \(2N\) samples. The one-dimensional Float DWT is defined by the following pair of analysis filter operations:
\[ C_j = \sum_{n=-4}^{4} h_n x_{2^{j+n}}; \quad D_j = \sum_{n=-3}^{3} g_n x_{2^{j+1+n}}; \quad j = 0, 1, \ldots, N-1 \]  

(3)

The outputs \(C_j, D_j\) are referred to as the \textit{coefficients} of the wavelet transform.

**NOTE** – In equation 3 the filter taps are chosen so that \(C_j\) and \(D_j\) represent low-pass and high-pass outputs, respectively. That is, \(C_j\) is a smoothed (low-pass) version of the original signal, while \(D_j\) contains high-pass information.

At the beginning (left boundary) of the signal, equation 3 requires signal values with negative indices, and at the end (right boundary) of the signal, it requires signal values with indices exceeding \(2N-1\). These values are obtained from the following mirror symmetric signal extension:

\[ x_m = x_{-m} \quad \text{for} \quad m < 0; \quad x_{2N-1+m} = x_{2N-1-m} \quad \text{for} \quad m > 0 \]  

(4)

### 3.3.2 9/7 INTEGER TRANSFORM

**NOTE** – An alternative transform in this Recommended Standard is a non-linear approximation to a 9/7 DWT. The non-linearity is introduced as the result of round-off operations used for the sake of producing integer outputs from the decorrelation step. The non-linear transform can be used to achieve lossless compression.

Like the Float DWT defined in equation 3, the single-level, 1-d Integer DWT maps a signal vector (equation 2) to two sets of wavelet coefficients, one high-pass set, \(D_j\), and one low-pass set, \(C_j\), in accordance with equations 5 and 6. Special boundary filters are required at either end of the signal, and lead to the formulas given in equations 5 and 6 for \(j=0, j=N-2,\) and \(j=N-1\).

Equations 5 and 6 define the integer transform that shall be used with this Recommended Standard. Given input values of \(x_i\), the \(D_j\) values in equation 5 shall be computed first and used subsequently to compute \(C_j\) values in equation 6.

\[
\begin{align*}
D_0 &= x_1 - \left[ \frac{9}{16} (x_0 + x_2) - \frac{1}{16} (x_2 + x_4) + \frac{1}{2} \right] \\
D_j &= x_{2j+1} - \left[ \frac{9}{16} (x_{2j} + x_{2j+2}) - \frac{1}{16} (x_{2j-2} + x_{2j+4}) + \frac{1}{2} \right] \quad \text{for} \quad j = 1, \ldots, N-3 \\
D_{N-2} &= x_{2N-3} - \left[ \frac{9}{16} (x_{2N-4} + x_{2N-2}) - \frac{1}{16} (x_{2N-6} + x_{2N-2}) + \frac{1}{2} \right] \\
D_{N-1} &= x_{2N-1} - \left[ \frac{9}{8} x_{2N-2} - \frac{1}{8} x_{2N-4} + \frac{1}{2} \right]
\end{align*}
\]  

(5)
\[ C_0 = x_0 - \left[ -\frac{D_0}{2} + \frac{1}{2} \right] \]
\[ C_j = x_{2j} - \left[ -\frac{D_{j-1} + D_j}{4} + \frac{1}{2} \right] \quad \text{for } j = 1, \ldots, N-1 \] (6)

NOTE – Because of the rounding procedure, the recommended transform is, strictly speaking, non-linear and hence not truly a DWT. Nevertheless, a (non-linear) strict inverse transform exists (see 3.4.2).

3.4 INVERSE DWT

3.4.1 INVERSE 9/7 FLOAT TRANSFORM

3.4.1.1 Synthesis Filter Coefficients

The inverse 9/7 Float DWT uses two sets of synthesis filter coefficients:

\[ \left\{ q_{-3}, q_{-2}, q_{-1}, q_0, q_1, q_2, q_3 \right\} \]
\[ \left\{ p_{-4}, p_{-3}, p_{-2}, p_{-1}, p_0, p_1, p_2, p_3, p_4 \right\} \] (7)

The numerical values of the synthesis filter coefficients are specified in table 3-2.

Table 3-2: Synthesis Filter Coefficients for the 9/7 Filter

<table>
<thead>
<tr>
<th>(i)</th>
<th>Lowpass Filter (q_i)</th>
<th>Highpass Filter (p_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.788485616406</td>
<td>-0.852698679009</td>
</tr>
<tr>
<td>±1</td>
<td>0.418092273222</td>
<td>0.377402855613</td>
</tr>
<tr>
<td>±2</td>
<td>-0.040689417609</td>
<td>0.110624404418</td>
</tr>
<tr>
<td>±3</td>
<td>-0.064538882629</td>
<td>-0.023849465020</td>
</tr>
<tr>
<td>±4</td>
<td></td>
<td>-0.037828455507</td>
</tr>
</tbody>
</table>
3.4.1.2 Synthesis Filter Operations

The one-dimensional DWT shall be inverted by means of a pair of synthesis filtering operations:

\[
\begin{align*}
\begin{cases}
    x_{2j} = \sum_{n=-1}^{1} q_{2n} C_{j+n} + \sum_{n=2}^{1} P_{2n+1} D_{j+n} \\
    x_{2j+1} = \sum_{n=-1}^{3} q_{2n-1} C_{j+n} + \sum_{n=2}^{3} P_{2n} D_{j+n}
\end{cases}
\end{align*}
\]

\[(8)\]

For correct evaluation of equation 8 at the boundaries, the wavelet coefficient signals \(C_j, D_j\) must be extended as follows:

\[
\begin{align*}
    D_m &= D_{-m-1} \quad \text{for } m < 0; \quad D_{N-1-m} = D_{N-1-1-m} \quad \text{for } m > 0 \\
    C_m &= C_{-m} \quad \text{for } m < 0; \quad C_{N-1-m} = C_{N-1-m} \quad \text{for } m > 0
\end{align*}
\]

\[(9)\]

3.4.2 INVERSE INTEGER TRANSFORM

Like the inverse Float DWT defined in equation 8, the inverse Integer DWT maps two sets of wavelet coefficients, a low-pass set, \(C_j\), and a high-pass set, \(D_j\), back to a signal vector \(x_j\) in accordance with equations 10 and 11. Special boundary filters are required at either end of the data sets, leading to the formulas given in equations 10 and 11 for \(j=0, j=1, j=2N-3\), and \(j=2N-1\).

\[
\begin{align*}
    x_0 &= C_0 + \left[ -\frac{D_0}{2} + \frac{1}{2} \right] \\
    x_{2j} &= C_j + \left[ -\frac{D_{j-1} + D_j}{4} + \frac{1}{2} \right] \quad \text{for } j = 1, \ldots, N-1 \\
    x_1 &= D_0 + \left[ \frac{9}{16} (x_0 + x_2) - \frac{1}{16} (x_2 + x_4) + \frac{1}{2} \right] \\
    x_{2j+1} &= D_j + \left[ \frac{9}{16} (x_{2j} + x_{2j+2}) - \frac{1}{16} (x_{2j-2} + x_{2j+4}) + \frac{1}{2} \right] \quad \text{for } j = 1, \ldots, N-3 \\
    x_{2N-3} &= D_{N-2} + \left[ \frac{9}{16} (x_{2N-4} + x_{2N-2}) - \frac{1}{16} (x_{2N-6} + x_{2N-2}) + \frac{1}{2} \right] \\
    x_{2N-1} &= D_{N-1} + \left[ \frac{9}{8} x_{2N-2} - \frac{1}{8} x_{2N-4} + \frac{1}{2} \right]
\end{align*}
\]

\[(10)\]

\[(11)\]
In equation 10, the even indexed signal values $x_0, x_2, \ldots, x_{2N-2}$ shall be computed first from the DWT coefficients via equation 10. These reconstructed values shall then be utilized to compute odd-indexed signal values $x_1, x_3, \ldots, x_{2N-1}$ via equation 11.

### 3.5 TWO-DIMENSIONAL SINGLE-LEVEL DWT

Image decorrelation is accomplished using a two-dimensional DWT, which is performed by iterated application of the one-dimensional DWT. Viewing the image as a data matrix consisting of rows and columns of signal vectors, a single-level 2-d DWT shall be performed on the image in the following two steps in the following order:

a) the 1-d DWT shall be performed on each image row, producing a horizontally low-pass and a horizontally high-pass filtered intermediate data array, each half as wide as the original image array, as illustrated in figure 3-1(b);

b) the 1-d DWT shall be applied to each column of both intermediate data arrays to produce four subbands as shown in figure 3-1(c).

![Figure 3-1: 2-d DWT (One Level)](image)

Each of the four subband data arrays obtained is half as wide and half as tall as the original image array. In illustrations, these subbands are often shown arranged as one array which has the same size as the original image array (see figure 3-1(c)). Starting at the upper left and proceeding clockwise in figure 3-1(c), the four subbands are referred to as LL, HL, HH, LH.
3.6 INVERSE OF TWO-DIMENSIONAL DWT

The single-level 2-d DWT transform shall be inverted by repeated application of the 1-d inverse to columns and rows of the transformed data array in the reverse order to that in which the 1-d transforms were applied:

a) each column shall be inverted to produce the intermediate transformed data arrays:
   1) the 1-d DWT inverse shall be applied to columns of the LL and LH subbands to obtain the intermediate horizontal low-pass array of figure 3-1(b),
   2) the 1-d DWT inverse shall be applied to columns of the HL and HH subbands to obtain the intermediate horizontal high-pass array of figure 3-1(b);

b) the 1-d DWT inverse shall be applied to rows of the intermediate horizontal low-pass and horizontal high-pass arrays to recover the original image array.

This inverse procedure shall be used for both lossless and lossy decompression.

3.7 MULTI-LEVEL TWO-DIMENSIONAL DWT

To increase compression effectiveness, correlation remaining in the LL subband after the 2-d DWT decomposition is exploited by applying further levels of DWT decomposition to produce a multi-level 2-d DWT. This produces the pyramidal decomposition described in reference [B7].

This Recommended Standard specifies three levels of decomposition.

At each level, the 2-d DWT described in 3.5 shall be applied to the LL subband produced by the previous level of decomposition.

NOTE – Figure 3-2 illustrates a three-level 2-d DWT decomposition. At each level of decomposition, the LL subband from the previous level is decomposed, using a 2-d DWT, and is replaced with four new subbands. Each new subband is half the width and half the height of the LL subband from which it was computed. Each additional level of decomposition thus increases the number of subbands by three but leaves unchanged the total number of DWT coefficients used to represent the image data. Following \( n \) levels of 2-d DWT decomposition, the total number of subbands is therefore \( 3n+1 \). Following the recommendation of a three-level decomposition, ten subbands are generated. The subbands are typically shown arranged to form an array of the same dimensions as the original image, as is done in figure 3-2. Subscripts are added to LL, HL, HH, and LH to denote the level of decomposition.
Figure 3-2: Three-Level 2-d DWT Decomposition of an Image

3.8 INVERSE MULTI-LEVEL 2-D DWT

The inversion process of a multi-level DWT shall be as follows:

a) the four subbands of highest level, LL₃, LH₃, HL₃, HH₃, shall be inverted using an inverse single-level 2-d DWT to yield the single subband LL₂, which then replaces the higher-level subbands in the transform data matrix;

b) the four subbands LL₂, LH₂, HL₂, HH₂ shall be inverted to yield the single subband LL₁, which again replaces the higher-level subbands in the transform data matrix;

c) a final single-level 2-d inverse DWT shall be applied to subbands LL₁, LH₁, HL₁, HH₁ to reproduce the original image.
3.9 SUBBAND WEIGHTS

3.9.1 The Bit-Plane Encoder (BPE) described in section 4 shall be used to encode the subbands produced by the 2-d DWT decomposition.

3.9.2 For the integer transform, all subband coefficients shall be multiplied by their respective weight factors before encoding. This multiplication is not done for the float transform.

3.9.3 Standard weights for use in multiplying subband coefficients of the integer DWT are given in table 3-3.

3.9.4 A user-defined set of weights may be substituted if a user wishes to emphasize data in some subbands over others. This option is flagged in Segment Header Part 4 (4.2.5).

Table 3-3: Standard Subband Weights for 9/7 Integer DWT

<table>
<thead>
<tr>
<th>Subband</th>
<th>HH(_1)</th>
<th>HL(_1), LH(_1)</th>
<th>HH(_2)</th>
<th>HL(_2), LH(_2)</th>
<th>HH(_3)</th>
<th>HL(_3), LH(_3)</th>
<th>LL(_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight factor</td>
<td>2(^0)</td>
<td>2(^1)</td>
<td>2(^1)</td>
<td>2(^2)</td>
<td>2(^2)</td>
<td>2(^3)</td>
<td>2(^3)</td>
</tr>
</tbody>
</table>

NOTE – The weight factors are chosen to be powers of two to allow scaling to be performed using bit-shift operations. For example, table 3-3 indicates that after applying a three-level 2-d DWT using the integer transform, each of the resulting DWT coefficients in the LH\(_2\) subband should be multiplied by a factor of 2\(^2\), which can be accomplished by two left bit-shift operations.
4 THE BIT PLANE ENCODER (BPE)

4.1 OVERVIEW

Following the DWT, the wavelet coefficients are either rounded to the nearest integer (when the floating-point transform has been used), or scaled using the weighting factors described in 3.9 (when the integer transform has been used). When scaling is performed, one or more of the least-significant bits in many of the subbands are necessarily all zeros. The BPE process takes this into account, i.e., bits that must be all zeros because of the scaling process are not encoded in the BPE. In the description in this section, the coefficient values mentioned refer to values after any scaling or rounding operation. The notation BitShift($\Gamma$) is used to indicate the number of least significant bits in each coefficient of the subband $\Gamma$ that are necessarily zero as a result of the subband scaling operation. When the 9/7 Float DWT is used, sub-bands are not scaled, thus BitShift equals zero for each subband.

The Bit Plane Encoder (BPE) processes wavelet coefficients in groups of 64 coefficients referred to as *blocks*. An example of a block is illustrated in figure 4-1 as comprised of shaded pixels. A block loosely corresponds to a localized region in the original image.

![Figure 4-1: Schematic of Wavelet-Transformed Image](image)

Information pertaining to a block of coefficients is jointly encoded by the BPE. A block consists of a single coefficient from the LL3 subband, referred to as the *DC coefficient*, and 63 *AC coefficients*. The AC coefficients in a block are arranged into three *families*, $F_0$, $F_1$ and $F_2$. Figure 4-1 illustrates a single block of coefficients and the family structure.
Each family $F_i$ in the block has one parent coefficient, $p_i$, a set $C_i$ of four children coefficients, and a set $G_i$ of sixteen grandchildren coefficients. The grandchildren in family $F_i$ are further partitioned into groups numbered $j=0,1,2,3$, denoted $H_{ij}$, as illustrated in figure 4-1. This structure is used for jointly encoding information pertaining to groups of coefficients in the block, as described in 4.5.

A wavelet coefficient is identified by its coordinates within its subband. Thus coordinates $(r, c)$ indicate the wavelet coefficient in row $r$, column $c$ within the subband, with the upper left pixel in a subband having coordinates $(0,0)$.

The DC coefficient for each block is a single coefficient from the LL3 subband. The coordinates for the other coefficients in the block can be determined from the coordinates of the DC coefficient. For a block with DC coefficient with coordinates $(r, c)$ within the LL3 subband, table 4-1 lists the coordinates for the AC coefficients, within their respective subbands of origin. Table 4-2 gives the subband of origin for each type of coefficient, determined by family and family member type. The relationship between all coefficients in a block is further illustrated graphically in figure 4-1. Blocks shall be processed by the Bit Plane Encoder consecutively in the raster scan in the order in which their corresponding DC coefficients occur in LL3: row by row, each row being processed from left to right.

### Table 4-1: Within-Subband Coordinates for Coefficients in a Single Family

<table>
<thead>
<tr>
<th>Coefficient Group in Family $i$</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent, $p_i$</td>
<td>$(r, c)$</td>
</tr>
<tr>
<td>Children group, $C_i$</td>
<td>$(2r, 2c), (2r, 2c+1), (2r+1, 2c), (2r+1, 2c+1)$</td>
</tr>
<tr>
<td>Grandchildren group, $H_{i0}$</td>
<td>$(4r, 4c), (4r, 4c+1), (4r+1, 4c), (4r+1, 4c+1)$</td>
</tr>
<tr>
<td>Grandchildren group, $H_{i1}$</td>
<td>$(4r, 4c+2), (4r, 4c+3), (4r+1, 4c+2), (4r+1, 4c+3)$</td>
</tr>
<tr>
<td>Grandchildren group, $H_{i2}$</td>
<td>$(4r+2, 4c), (4r+2, 4c+1), (4r+3, 4c), (4r+3, 4c+1)$</td>
</tr>
<tr>
<td>Grandchildren group, $H_{i3}$</td>
<td>$(4r+2, 4c+2), (4r+2, 4c+3), (4r+3, 4c+2), (4r+3, 4c+3)$</td>
</tr>
</tbody>
</table>

### Table 4-2: Subband of Origin for AC Coefficients

<table>
<thead>
<tr>
<th></th>
<th>Family 0</th>
<th>Family 1</th>
<th>Family 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent</td>
<td>HL$_3$</td>
<td>LH$_3$</td>
<td>HH$_3$</td>
</tr>
<tr>
<td>Children</td>
<td>HL$_2$</td>
<td>LH$_2$</td>
<td>HH$_2$</td>
</tr>
<tr>
<td>Grandchildren</td>
<td>HL$_1$</td>
<td>LH$_1$</td>
<td>HH$_1$</td>
</tr>
</tbody>
</table>
A segment is defined as a group of \( S \) consecutive blocks. Coding of DWT coefficients proceeds segment-by-segment and each segment is coded independently of the others. \( S \) can be assigned to any value between \( 16 \leq S \leq 2^{20} \), except for the last segment of an image, for which \( S \) can be assigned to any value between \( 1 \leq S \leq 2^{20} \). The value might be chosen based on the memory available to store the segment. When multiple image frames are transmitted, the coding of each new frame starts with a new segment, i.e., a single segment must not contain coded data from two separate frames.

A segment of blocks is further partitioned into gaggles. Each gaggle consists of 16 blocks, except for possibly the last gaggle in a segment, which contains \( S \mod 16 \) blocks when \( S \) is not a multiple of 16.

DC coefficients are represented using two’s-complement representation. Let \( c_m \) denote the \( m^{th} \) DC coefficient in a segment, i.e., the DC coefficient of the \( m^{th} \) block in a segment. The number of bits needed to represent \( c_m \) in two’s-complement representation is given in equation 12:

\[
1 + \left\lceil \log_2|c_m| \right\rceil, \quad \text{if } c_m < 0
\]
\[
1 + \left\lceil \log_2(1+c_m) \right\rceil, \quad \text{if } c_m \geq 0
\]

Within a segment, BitDepthDC is defined as the maximum of this value over all DC coefficients (i.e., all values of \( m \)) in the segment. Each DC coefficient in the segment is represented using BitDepthDC bits, in two’s-complement representation.

An AC coefficient is represented using the binary representation of the magnitude of the coefficient, along with a bit indicating the sign when the coefficient is nonzero. BitDepthAC\_Block\_m in equation 13 denotes the maximum number of bits needed to specify the magnitude of any AC coefficient in the \( m^{th} \) block.

\[
\text{BitDepthAC\_Block}_m = \left\lceil \log_2(1 + \max_x|\langle x \rangle|) \right\rceil
\]

where the maximization is over all AC coefficients \( x \) in the block.

For each segment, the BPE computes BitDepthAC, which denotes the maximum value of BitDepthAC\_Block\_m for the segment:

\[
\text{BitDepthAC} = \max_{m=0, \ldots, S-1} \text{BitDepthAC\_Block}_m
\]

The BPE successively encodes bit planes of coefficient magnitudes in a segment, inserting AC coefficient sign values at appropriate points in the encoded data stream. Bit plane \( b \) consists of the \( b^{th} \) bit of the two’s-complement integer representation of each DC coefficient, and the \( b^{th} \) bit of the binary integer representation of the magnitude of each AC coefficient. Here, bit plane index \( b=0 \) corresponds to the least significant bit. The BPE proceeds from most-significant bit to least significant bit, thus \( b \) decreases from one bit plane to the next, beginning with \( b = \text{BitDepthAC}-1 \), and ending with \( b=0 \). DWT coefficient resolution effectively improves by a factor of 2 as encoding proceeds from one bit plane to the next. The bit plane coding process is described in 4.5.
Figure 4-2(a) gives an overview of the structure of any single coded segment. Within a segment, header information is encoded. Then quantized DC coefficients from the blocks are encoded. Then AC bit depths are encoded. Then DWT coefficient blocks are encoded, one bit plane at a time, proceeding from the most significant to the least significant bit plane. The coding of a single bit plane is performed in several stages, and the resulting order of code is illustrated in figure 4-2(b). E.g., parent coefficients are coded in stage 1 for all blocks of the segment before encoding child coefficients in stage 2. The resulting encoded bit stream constitutes an embedded data format that provides progressive transmission.

(a) With All Bit Planes

<table>
<thead>
<tr>
<th>Segment Header (see 4.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial coding of DC coefficients (see 4.3)</td>
</tr>
<tr>
<td>Coded AC coefficient bit depths (see 4.4)</td>
</tr>
<tr>
<td>Coded bit plane $b = \text{BitDepthAC}-1$ (see 4.5)</td>
</tr>
<tr>
<td>Coded bit plane $b = \text{BitDepthAC}-2$ (see 4.5)</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>Coded bit plane $b = 0$ (see 4.5)</td>
</tr>
</tbody>
</table>

(b) Within One Bit Plane

Figure 4-2: Overview of the Structure of a Coded Segment

The tradeoff between reconstructed image quality and compressed data volume for each segment is controlled by specifying the maximum number of bytes in each compressed segment, SegByteLimit, and a ‘quality’ limit. The quality limit constrains the amount of DWT coefficient information to be encoded, and is specified as a bit plane index and a stopping point within that bit plane, as described in 4.2. Compressed output for a segment is produced until the byte limit or quality limit is reached, whichever comes first.
The encoded bitstream for a segment can be further truncated (or, equivalently, coding can be terminated early) at any point to further reduce the data rate, at the price of reduced image quality for the corresponding segment.

The remainder of this section describes each component of the coded segment. Subsection 4.2 describes the segment header. Subsection 4.3 describes the initial coding of DC coefficients. Subsection 4.4 describes the coded sequence of BitDepthAC_Block\(_m\) values. Subsection 4.5 describes the coding process for a bit plane of the segment.

4.2 SEGMENT HEADER

4.2.1 GENERAL

4.2.1.1 Each compressed segment shall begin with a segment header consisting of the following parts in the following order:

a) Part 1 (three or four bytes, mandatory—see 4.2.2);

b) Part 2 (five bytes, optional—see 4.2.3);

c) Part 3 (three bytes, optional—see 4.2.4);

d) Part 4 (eight bytes, optional—see 4.2.5).

4.2.1.2 By CCSDS convention, all reserved bits in each header part shall be set to ‘zero’.

4.2.1.3 Optional header parts may be omitted when the parameters described in a part can be determined without that part, e.g., when those parameters are set to known fixed values for an entire mission.

NOTE – Figure 4-3 gives an overview of the header structure and table 4-3 provides a high-level description of the functionality of each part of the header.

<table>
<thead>
<tr>
<th>Part 1A</th>
<th>Part 1B</th>
<th>Part 2</th>
<th>Part 3</th>
<th>Part 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 bytes</td>
<td>1 byte</td>
<td>5 bytes</td>
<td>3 bytes</td>
<td>8 bytes</td>
</tr>
</tbody>
</table>

Figure 4-3: Overview of Segment Header Structure When All Parts Are Included
Table 4-3: Summary of Segment Header Functionality

<table>
<thead>
<tr>
<th>Header Part</th>
<th>Length (Bytes)</th>
<th>Status</th>
<th>Description of Contents</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1A</td>
<td>3</td>
<td>Mandatory</td>
<td>Flags first and last segments of image; indicates which optional header parts are included; encodes information that typically changes from segment-to-segment.</td>
<td>4.2.2</td>
</tr>
<tr>
<td>Part 1B</td>
<td>1</td>
<td>Mandatory for the last segment of image, not included otherwise</td>
<td>Specifies number of ‘padding’ rows to be deleted after image reconstruction.</td>
<td>4.2.2</td>
</tr>
<tr>
<td>Part 2</td>
<td>5</td>
<td>Optional</td>
<td>Specifies limits on compressed bytes per segment and image quality.</td>
<td>4.2.3</td>
</tr>
<tr>
<td>Part 3</td>
<td>3</td>
<td>Optional</td>
<td>Coding options including number of blocks per segment.</td>
<td>4.2.4</td>
</tr>
<tr>
<td>Part 4</td>
<td>8</td>
<td>Optional</td>
<td>Image and compression parameters that must be fixed for an image.</td>
<td>4.2.5</td>
</tr>
</tbody>
</table>

NOTE – The mandatory first part of the header includes values of segment information that change from segment-to-segment. The optional second part of the header specifies limits on the number of compressed bytes in a segment and the limits on the fidelity with which DWT coefficients are encoded. This part might be included at the start of an image or application session, or at the beginning of each coded segment for variable output rate control. The optional third part of the header specifies information that is typically fixed for each image or application session, but is allowed to change with each segment. In a typical application, this part might be included at the beginning of each image, but not included for each segment. The fourth part of the header specifies parameters that must be fixed for an entire image.

4.2.2 SEGMENT HEADER PART 1

4.2.2.1 General

Segment Header Part 1 consists of two subparts:

a) Part 1A (3 bytes, mandatory for all segments);

b) Part 1B (1 byte, mandatory for the last segment, otherwise not used).

NOTE – Table 4-4 summarizes the contents of Segment Header Part 1. Figure 4-4 illustrates the Segment Header Part 1 structure.
### Table 4-4: Contents of Segment Header Part 1

<table>
<thead>
<tr>
<th>Part 1A</th>
<th>Field</th>
<th>Width (bits)</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>StartImgFlag</td>
<td>1</td>
<td>Flags initial segment in an image</td>
<td>1: first segment in image 0: continuation segment in image</td>
</tr>
<tr>
<td></td>
<td>EndImgFlag</td>
<td>1</td>
<td>Flags final segment in an image</td>
<td>1: last segment in image 0: otherwise</td>
</tr>
<tr>
<td></td>
<td>SegmentCount</td>
<td>8</td>
<td>Segment counter value</td>
<td>segment count value (mod 256), encoded as an unsigned binary integer</td>
</tr>
<tr>
<td></td>
<td>BitDepthDC</td>
<td>5</td>
<td>Number of bits needed to represent DC coefficients in 2's complement representation</td>
<td>value of BitDepthDC encoded as an unsigned binary integer</td>
</tr>
<tr>
<td></td>
<td>BitDepthAC</td>
<td>5</td>
<td>Number of bits needed to represent absolute value of AC coefficients in unsigned integer representation</td>
<td>value of BitDepthAC encoded as an unsigned binary integer</td>
</tr>
<tr>
<td></td>
<td>Reserved</td>
<td>1</td>
<td>Reserved for future use</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Part2Flag</td>
<td>1</td>
<td>Indicates presence of Part 2 header</td>
<td>1: Part 2 of header present 0: Part 2 of header absent</td>
</tr>
<tr>
<td></td>
<td>Part3Flag</td>
<td>1</td>
<td>Indicates presence of Part 3 header</td>
<td>1: Part 3 of header present 0: Part 3 of header absent</td>
</tr>
<tr>
<td></td>
<td>Part4Flag</td>
<td>1</td>
<td>Indicates presence of Part 4 header</td>
<td>1: Part 4 of header present 0: Part 4 of header absent</td>
</tr>
<tr>
<td></td>
<td>PadRows</td>
<td>3</td>
<td>Number of ‘padding’ rows to delete after inverse DWT</td>
<td>Value encoded as unsigned binary integer</td>
</tr>
<tr>
<td></td>
<td>Reserved</td>
<td>5</td>
<td>Reserved for future use</td>
<td>00000</td>
</tr>
</tbody>
</table>

### Figure 4-4: Segment Header Part 1 When Part 1B Is Included
4.2.2.2 Segment Header Part 1A

4.2.2.2.1 Bit 0 of Segment Header Part 1A shall contain the Start Image flag (StartImgFlag). The Start Image flag shall be used to indicate whether the segment corresponds to the start of an image:

- ‘1’ shall indicate that the segment is the first segment in the image;
- ‘0’ shall indicate that the segment is not the first segment.

4.2.2.2.2 Bit 1 of Segment Header Part 1A shall contain the End Image flag (EndImgFlag). The End Image flag shall be used to indicate whether the segment is the last segment in the image:

- ‘1’ shall indicate that the segment is the last segment in the image, in which case Segment Header Part 1B must be included;
- ‘0’ shall indicate that the segment is not the last segment, in which case Segment Header Part 1B shall not be included.

4.2.2.2.3 Bits 2-9 of Segment Header Part 1A shall provide the segment counter (SegmentCount):

a) the eight-bit segment counter shall be encoded as an unsigned binary integer;

b) the segment counter shall equal zero for the first segment of an image and shall increment by one for each subsequent segment;

c) the segment counter shall wrap back to zero and resume incrementing after reaching a value of 255.

NOTE – This counter provides an index of segments, modulo 256, within the image. Absolute segment counting can be implemented using the CCSDS Packet Sequence Count as described in reference [B3].

4.2.2.2.4 Bits 10-14 shall contain the BitDepthDC field. The value of the five-bit BitDepthDC field shall be encoded as an unsigned binary integer and shall equal the number of bits needed to represent DC coefficients for the segment in 2’s complement representation.

4.2.2.2.5 Bits 15-19 shall contain the BitDepthAC field. The value of the five-bit BitDepthAC variable shall be encoded as an unsigned binary integer and shall equal the number of bits needed to represent the absolute value of AC coefficients in unsigned integer representation (see 4.4).

4.2.2.2.6 Bit 20 is reserved for future use by the CCSDS and shall be set to ‘0’.
4.2.2.2.7 Bit 21 of Segment Header Part 1A shall contain the Part 2 flag (Part2Flag) and shall indicate the presence or absence of optional Segment Header Part 2:

- ‘1’ shall indicate that Segment Header Part 2 is present;
- ‘0’ shall indicate that Segment Header Part 2 is absent.

4.2.2.2.8 Bit 22 of Segment Header Part 1A shall contain the Part 3 flag (Part3Flag) and shall indicate the presence or absence of optional Segment Header Part 3:

- ‘1’ shall indicate that Segment Header Part 3 is present;
- ‘0’ shall indicate that Segment Header Part 3 is absent.

4.2.2.2.9 Bit 23 of Segment Header Part 1A shall contain the Part 4 flag (Part4Flag) and shall indicate the presence or absence of optional Segment Header Part 4:

- ‘1’ shall indicate that Segment Header Part 4 is present;
- ‘0’ shall indicate that Segment Header Part 4 is absent.

4.2.2.3 Segment Header Part 1B

4.2.2.3.1 When EndImgFlag (4.2.2.2.2) in Segment Header Part 1A is set to ‘1’, indicating that the segment is the last segment of the image, Segment Header Part 1B shall be included.

4.2.2.3.2 Bits 0-2 of Segment Header Part 1B shall contain the PadRows field. The value of the three-bit PadRows field shall be encoded as an unsigned binary integer and shall equal the number of ‘padding’ rows to be deleted (if any) after the inverse DWT is performed (see 3.2).

4.2.2.3.3 Bits 3-7 of Segment Header Part 1B are reserved for future use by the CCSDS and shall be set to ‘0’.

4.2.3 SEGMENT HEADER PART 2

4.2.3.1 General

If used, the optional Segment Header Part 2 shall specify output options that control the tradeoff between compressed data volume and reconstructed image quality for a segment.

NOTE – Table 4-5 summarizes the contents of Segment Header Part 2. Figure 4-5 illustrates the Segment Header Part 2 structure.
Table 4-5: Contents of Segment Header Part 2

<table>
<thead>
<tr>
<th>Field</th>
<th>Width (bits)</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>SegByteLimit</td>
<td>27</td>
<td>Maximum number of compressed bytes in a segment.</td>
<td>value (mod $2^{27}$) encoded as an unsigned integer</td>
</tr>
<tr>
<td>DCStop</td>
<td>1</td>
<td>Indicates whether compressed output stops after coding of quantized DC coefficients (4.3).</td>
<td>1: Stop coding after coding quantized DC coefficient information (see 4.3.2) and additional DC bit planes (see 4.3.3) 0: Coding stop determined by BitPlaneStop and StageStop values</td>
</tr>
<tr>
<td>BitPlaneStop</td>
<td>5</td>
<td>Unused when DCStop = 1. When DCStop = 0, indicates limit on coding of DWT coefficient bit planes. When BitPlaneStop = b and StageStop = s, compressed output stops once stage s of bit plane b has been completed (see 4.5), unless coding stops earlier in the segment because of the segment byte limit (SegByteLimit).</td>
<td>Bit plane index value BitPlaneStop encoded as an unsigned integer</td>
</tr>
<tr>
<td>StageStop</td>
<td>2</td>
<td></td>
<td>00: stage 1 (see 4.5.3) 01: stage 2 (see 4.5.3) 10: stage 3 (see 4.5.3) 11: stage 4 (see 4.5.4)</td>
</tr>
<tr>
<td>UseFill</td>
<td>1</td>
<td>Specifies whether fill bits will be used to produce SegByteLimit bytes in each segment.</td>
<td>1: fills bits used whenever needed to produce SegByteLimit bytes in each segment 0: fills bits not allowed</td>
</tr>
<tr>
<td>Reserved</td>
<td>4</td>
<td>Reserved for future use.</td>
<td>0000</td>
</tr>
</tbody>
</table>

Figure 4-5: Segment Header Part 2
4.2.3.2   Contents of Segment Header Part 2

4.2.3.2.1   Bits 0-26 of Segment Header Part 2 shall contain the SegByteLimit field:

a) the value of the SegByteLimit field shall be encoded as an unsigned binary integer and shall equal the maximum number of bytes that can be used in a segment, including bytes used for the header;

NOTE – A SegByteLimit value less than 9 before the last segment and 10 for the last segment would not be permitted because of the combined number of bytes in the header Parts 1 and 2. The value of SegByteLimit must be an integer multiple of the word size specified by CodeWordLength in 4.2.5.2.7.

b) the value shall apply to the current segment and all subsequent segments until a new value of SegByteLimit is encoded in Segment Header Part 2 of a later segment header.

4.2.3.2.2   Bit 27 of Segment Header Part 2 shall contain the DCStop flag. The DCStop flag shall indicate the end of compressed output:

a) ‘1’ shall indicate that compressed output terminates once the quantized DC coefficient values (see 4.3.2) and additional bit planes of DC coefficients (see 4.3.3) are encoded;

b) ‘0’ shall indicate that end of compressed output shall be determined by a bit plane index (BitPlaneStop) and a coding stage within the coding of that bit plane (StageStop).

NOTE – Coded segment data stops at the indicated point in the BPE coding process, or when the segment byte limit is reached, whichever comes first. Compressed data output always includes at least the coded quantized DC coefficients values and any additional DC bit planes of DC coefficients (4.3.2 and 4.3.3) unless the value of SegByteLimit is too small to allow it.

4.2.3.2.3   Bits 28-32 of Segment Header Part 2 shall contain the BitPlaneStop field:

a) the 5-bit BitPlaneStop field shall be encoded as an unsigned binary integer and shall indicate the limit on coding of DWT coefficient bit planes;

b) when DCStop is set to ‘0’, the value of BitPlaneStop shall equal the index value of the bit plane in which coding stops, unless coding stops earlier because SegByteLimit is reached first;

c) when DCStop is set to ‘1’ the value of BitPlaneStop and the value of StageStop shall be ignored.

NOTE – When the user supplied BitPlaneStop value is greater than BitDepthAC-1, compression will terminate after completing coding of the quantized DC coefficient values and any additional DC bit planes (4.3.2 and 4.3.3) unless the SegByteLimit is too small to allow it.
4.2.3.2.4 Bits 33-34 of Segment Header Part 2 shall contain the StageStop field. The two-bit StageStop field shall indicate the stage at which the coding stops in the bit plane indicated by BitPlaneStop:

- ‘00’ shall indicate stage 1 (see 4.5.3);
- ‘01’ shall indicate stage 2 (see 4.5.3);
- ‘10’ shall indicate stage 3 (see 4.5.3);
- ‘11’ shall indicate stage 4 (see 4.5.4).

NOTE – Compression is limited entirely by the segment byte limit when DCStop is 0, BitPlaneStop is 0, and StageStop is set to ‘11’ (i.e., stage 4). In this case, lossless compression is achieved for a segment when the Integer DWT is used and the compressed data for the segment requires less than SegByteLimit bytes.

4.2.3.2.5 Bit 35 of Segment Header Part 2 shall contain the UseFill field, which shall indicate whether fill bits will be used to produce SegByteLimit bytes in each segment:

a) if the value of UseFill field is ‘1’, fill bits are used whenever needed to produce SegByteLimit bytes in each segment;

b) if the value of UseFill field is ‘0’:

1) fill bits are not used to produce SegByteLimit bytes, and the number of compressed bytes in a segment may be less than SegByteLimit bytes;

2) fill bits are used to fill to the next word boundary, when the stopping point (according to the specified values of DCStop, BitPlaneStop, StageStop) is reached before the byte limit specified in SegByteLimit;

NOTE – A word corresponds to the unit in which the compressor produces output, which may be a single byte, two bytes, or four bytes. For example, a compressor that produces 4 output bytes at a time might include as many as 31 fill bits in the last word of the compressed segment. The word length of the compressor is signaled by the field CodeWordLength in Segment Header Part 4 (see 4.2.5).

c) fill bits shall be set to ‘all zeros’.

4.2.3.2.6 Bits 36-39 of Segment Header Part 2 are reserved for future use by the CCSDS and shall be set to ‘all zeros’.
4.2.4  SEGMENT HEADER PART 3

4.2.4.1  General

4.2.4.1.1  If used, Segment Header Part 3 shall specify the segment size in terms of number of blocks, \( S \), defined in 4.1 and indicate whether the heuristic parameter selection approach (4.3.2) is used in the coding of quantized DC coefficients (see 4.3) and/or AC coefficient bit depths (see 4.4).

4.2.4.1.2  The information encoded in this part of the header is permitted to change with each segment.

NOTE – Table 4-6 summarizes the contents of Segment Header Part 3. Figure 4-6 illustrates the Segment Header Part 3 structure.

Table 4-6: Contents of Segment Header Part 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Width (bits)</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>20</td>
<td>segment size in blocks</td>
<td>Value encoded, mod ( 2^{20} ), as an unsigned binary integer</td>
</tr>
<tr>
<td>OptDCSelect</td>
<td>1</td>
<td>Specifies whether optimum or heuristic method is used to select value of ( k ) parameter for coding quantized DC coefficient values (see 4.3.2)</td>
<td>1: optimum selection of ( k ) 0: heuristic selection of ( k )</td>
</tr>
<tr>
<td>OptACSelect</td>
<td>1</td>
<td>Specifies whether optimum or heuristic method is used to select value of ( k ) parameter for coding BitDepthAC (see 4.4)</td>
<td>1: optimum selection of ( k ) 0: heuristic selection of ( k )</td>
</tr>
<tr>
<td>Reserved</td>
<td>2</td>
<td>Reserved</td>
<td>00</td>
</tr>
</tbody>
</table>

Figure 4-6: Segment Header Part 3
4.2.4.2 Segment Header Part 3 Contents

4.2.4.2.1 Bits 0-19 of Segment Header Part 3 shall contain the Segment Size field. The 20-bit Segment Size field shall be encoded as an unsigned binary integer and shall equal \( S \mod 2^{20} \); \( S \) shall be the number of blocks that make up the segment.

4.2.4.2.2 Bit 20 of Segment Header Part 3 shall contain the OptDCSelect field. The OptDCSelect field shall indicate the method used to select the value of the \( k \) parameter for coding quantized DC coefficient values (see 4.3.2.11):

- ‘1’ shall indicate optimum selection of \( k \);
- ‘0’ shall indicate heuristic selection of \( k \).

4.2.4.2.3 Bit 21 of Segment Header Part 3 shall contain the OptACSelect field. The OptACSelect field shall indicate the method used to select the value of the \( k \) parameter for coding AC coefficient bit depths (see 4.4):

- ‘1’ shall indicate optimum selection of \( k \);
- ‘0’ shall indicate heuristic selection of \( k \).

4.2.4.2.4 Bits 22-23 of Segment Header Part 3 are reserved for future use by the CCSDS and shall be set to ‘all zeros’.

4.2.5 SEGMENT HEADER PART 4

4.2.5.1 General

If used, optional Segment Header Part 4 shall provide information that does not change within an image, and that might often be fixed for an application session. This includes DWT type, optional user-defined weights for scaling the subbands, information about the input image, and coded word length.

NOTE – Table 4-7 summarizes the contents of Segment Header Part 4. Figure 4-7 illustrates the Segment Header Part 4 structure.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Width (bits)</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWTtype</td>
<td>1</td>
<td>Specifies DWT type</td>
<td>0: Float DWT 1: Integer DWT</td>
</tr>
<tr>
<td>Reserved</td>
<td>2</td>
<td>Reserved for future use</td>
<td>00</td>
</tr>
<tr>
<td>SignedPixels</td>
<td>1</td>
<td>Specifies whether input pixel values are signed or unsigned quantities</td>
<td>0: unsigned 1: signed</td>
</tr>
<tr>
<td>PixelBitDepth</td>
<td>4</td>
<td>Specifies the input pixel bit depth</td>
<td>Specifies values from 1 to 16 by mod(16) of 1, 2, 3, ..., 16</td>
</tr>
<tr>
<td>ImageWidth</td>
<td>20</td>
<td>image width in pixels</td>
<td>Value encoded, mod 2²⁰, as an unsigned binary integer</td>
</tr>
<tr>
<td>TransposeImg</td>
<td>1</td>
<td>Indicates whether entire image should be transposed after reconstruction</td>
<td>0: do not transpose image 1: transpose image</td>
</tr>
<tr>
<td>CodeWordLength</td>
<td>2</td>
<td>Indicates the coded word length</td>
<td>00: 8-bit word 01: 16-bit word 10: 24-bit word 11: 32-bit word</td>
</tr>
<tr>
<td>Reserved</td>
<td>1</td>
<td>Reserved for future use</td>
<td>0</td>
</tr>
<tr>
<td>CustomWtFlag</td>
<td>1</td>
<td>Indicates if weights in 3.9 used or user defined</td>
<td>0: weights in 3.9 used 1: user-defined weights</td>
</tr>
<tr>
<td>CustomWtHH₁</td>
<td>2</td>
<td>Weight of HH₁ subband</td>
<td>00: weight = 2⁰ 01: weight = 2¹ 10: weight = 2² 11: weight = 2³</td>
</tr>
<tr>
<td>CustomWtHL₁</td>
<td>2</td>
<td>Weight of HL₁ subband</td>
<td>(These fields are set to 00 when CustomWtFlag is 0)</td>
</tr>
<tr>
<td>CustomWtLH₁</td>
<td>2</td>
<td>Weight of LH₁ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtHH₂</td>
<td>2</td>
<td>Weight of HH₂ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtHL₂</td>
<td>2</td>
<td>Weight of HL₂ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtLH₂</td>
<td>2</td>
<td>Weight of LH₂ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtHH₃</td>
<td>2</td>
<td>Weight of HH₃ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtHL₃</td>
<td>2</td>
<td>Weight of HL₃ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtLH₃</td>
<td>2</td>
<td>Weight of LH₃ subband</td>
<td></td>
</tr>
<tr>
<td>CustomWtLL₃</td>
<td>2</td>
<td>Weight of LL₃ subband</td>
<td></td>
</tr>
<tr>
<td>Reserved</td>
<td>11</td>
<td>Reserved for future</td>
<td>000000000000</td>
</tr>
</tbody>
</table>
4.2.5.2 Segment Header Part 4 Contents

4.2.5.2.1 Bit 0 of Segment Header Part 4 shall contain the DWT Type field (DWTtype) and shall indicate the DWT type as follows:

- ‘0’ shall indicate Float DWT;
- ‘1’ shall indicate Integer DWT.

4.2.5.2.2 Bits 1-2 of Segment Header Part 4 are reserved for future use by the CCSDS and shall be set to ‘all zeros’.

4.2.5.2.3 Bit 3 of Segment Header Part 4 shall contain the Signed Pixels field (SignedPixels) and shall indicate whether input pixel values are signed or unsigned quantities:

- ‘0’ shall indicate unsigned;
- ‘1’ shall indicate signed.

4.2.5.2.4 Bits 4-7 of Segment Header Part 4 shall contain the Pixel Bit Depth field (PixelBitDepth). The four-bit Pixel Bit Depth field shall be encoded as an unsigned binary integer and shall equal the pixel bit depth mod 16.

4.2.5.2.5 Bits 8-27 of Segment Header Part 4 shall contain the Image Width field (ImageWidth). The 20-bit Image Width field shall be encoded as an unsigned binary integer and shall equal the width of the image in pixels mod $2^{20}$.

4.2.5.2.6 Bit 28 of Segment Header Part 4 shall contain the Transpose Image field (TransposeImg) and shall indicate whether the entire image should be transposed after reconstruction:

- ‘0’ shall indicate that the image should not be transposed;
- ‘1’ shall indicate that the image should be transposed.
4.2.5.2.7 Bits 29-30 of Segment Header Part 4 shall contain the Code Word Length field (CodeWordLength) and shall indicate the length of the code word as follows:
   - ‘00’: 8-bit word;
   - ‘01’: 16-bit word;
   - ‘10’: 24-bit word;
   - ‘11’: 32-bit word.

4.2.5.2.8 Bit 31 of Segment Header Part 4 is reserved for future use by the CCSDS and shall be set to ‘0’.

4.2.5.2.9 Bit 32 of Segment Header Part 4 shall contain the Custom Weights flag (CustomWtFlag) and shall indicate whether standard (3.9) or user-defined weights are used:
   - ‘0’ shall indicate that the subband weights defined in 3.9 are used;
   - ‘1’ shall indicate that user-defined subband weights are used.

4.2.5.2.10 Bits 33-52 of Segment Header Part 4 shall provide 10 two-bit custom subband weight fields for each of the subbands HH₁–LL₃:
   a) the custom subband weight fields shall be ordered as follows:
      - bits 33-34: HH₁,
      - bits 35-36: HL₁,
      - bits 37-38: LH₁,
      - bits 39-40: HH₂,
      - bits 41-42: HL₂,
      - bits 43-44: LH₂,
      - bits 45-46: HH₃,
      - bits 47-48: HL₃,
      - bits 49-50: LH₃,
      - bits 51-52: LL₃;

   b) if CustomWtFlag is set to ‘1’, the values of the custom subband weight fields shall be set as follows:
      - ‘00’: weight = 2⁰,
      - ‘01’: weight = 2¹,
      - ‘10’: weight = 2²,
      - ‘11’: weight = 2³;

   c) if CustomWtFlag is set to ‘0’, the values of bits 33-52 shall be set to ‘all zeros’.

4.2.5.2.11 Bits 53-63 of Segment Header Part 2 are reserved for future use by the CCSDS and shall be set to ‘all zeros’.
4.3 INITIAL CODING OF DC COEFFICIENTS

4.3.1 GENERAL

4.3.1.1 The initial coding of DC coefficients in a segment is performed in two steps. The first step shall be to encode a quantized representation of the DC coefficients using the predictive scheme described in this subsection (see 4.3.2). The second step shall be to encode \( q \)-BitDepthAC additional bit planes of DC coefficients (see 4.3.3), when \( q > \text{BitDepthAC} \) (\( q \) will be described below). The remaining BitDepthAC bits of the DC coefficient in the segment shall be coded during stage 0 of the bit plane coding process of 4.5 as needed, when the bit plane index \( b \) satisfies \( b < q \).

4.3.1.2 The amount of quantization of DC coefficients performed in this coding step shall be determined by the dynamic range of the AC and DC coefficients in a segment via the integer parameter \( q' \), which is defined according to table 4-8.

<table>
<thead>
<tr>
<th>DC and AC Dynamic Range</th>
<th>( q' ) value</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>BitDepthDC ≤ 3</td>
<td>( q' = 0 )</td>
<td>DC dynamic range is very small: no quantization is performed – all DC coefficient information is encoded in this step.</td>
</tr>
<tr>
<td>BitDepthDC – ( (1 + \lfloor \text{BitDepthAC}/2 \rfloor) \leq 1 ) and BitDepthDC &gt; 3</td>
<td>( q' = \text{BitDepthDC} – 3 )</td>
<td>DC dynamic range is close to half the AC dynamic range: the 3 most significant bits of the DC coefficients are differentially coded at this step.</td>
</tr>
<tr>
<td>BitDepthDC – ( (1 + \lfloor \text{BitDepthAC}/2 \rfloor) &gt; 10 ) and BitDepthDC &gt; 3</td>
<td>( q' = \text{BitDepthDC} – 10 )</td>
<td>DC dynamic range is much higher than half the AC dynamic range: the 10 most significant bits of the DC coefficients are differentially coded at this step.</td>
</tr>
<tr>
<td>otherwise</td>
<td>( q' = 1 + \lfloor \text{BitDepthAC}/2 \rfloor )</td>
<td>DC dynamic range is moderately higher than half the AC dynamic range: the DC coefficient bits exceeding half the AC dynamic range are differentially coded in this step.</td>
</tr>
</tbody>
</table>
4.3.1.3 DC quantization factor \( q \) in equation 15 is defined as:

\[
q = \max(q', \text{BitShift}(\text{LL}_3))
\]  

(15)

to take into account the DC bit planes that are necessarily all zeros as a result of the subband scaling operation.

NOTE – The value of \( q \) indicates the number of least-significant bits in each DC coefficient that are not encoded in the quantized DC coefficient values. The value of \( q \) is intended to be small enough that a significant amount of correlation in the DC coefficients can be exploited in the compression of these quantized coefficients. At the same time, the value of \( q \) is intended to be large enough to ensure that the BPE does not spend a large number of bits coding the DC coefficients to very high resolution before encoding any AC coefficient information.

4.3.1.4 For quantization in the initial DC coding phase, the DC quantization factor shall be compared to the dynamic range BitDepthAC of the AC coefficients of the segment, following the rules defined in table 4-8.

NOTE – The DC refinement bits required for coding DC coefficients with more precision are coded later, in stage 0 of Bit Plane Coding, as described in 4.5.

4.3.1.5 Next, given a sequence of DC coefficients \( \{c_m: m=0,...,S-1\} \) in a segment, the BPE shall compute quantized coefficients

\[
c'_m = \left\lfloor \frac{c_m}{2^q} \right\rfloor
\]  

(16)

4.3.1.6 The quantized DC coefficients shall be encoded using the procedure described in 4.3.2, which effectively encodes the BitDepthDC - \( q \) most significant bits from each DC coefficient.

4.3.1.7 When \( q > \text{BitDepthAC} \), the \( q \) - BitDepthAC next most significant bits of each DC coefficient appear in the coded bitstream, as described in 4.3.3.

4.3.1.8 Remaining bits of DC coefficients appear in the coded bit stream as part of the bit plane coding procedure described in 4.5, except for the BitShift(\text{LL}_3) least significant bits of each DC coefficient, which must be zero because of coefficient scaling, and are thus not encoded.

4.3.2 CODING QUANTIZED DC COEFFICIENTS

4.3.2.1 The number of bits needed to represent each quantized DC coefficient \( c'_m \) is given in equation 17 as:

\[
N = \max \{\text{BitDepthDC} - q, 1\}
\]  

(17)

NOTE – The DC coefficient quantization rules in table 4-8 limit the value of \( N \) to be within 10, and thus coding options are defined only up to \( N = 10 \).
4.3.2.2 When \( N \) is 1, each quantized DC coefficient \( c'_m \) consists of a single bit. In this case, the coded quantized DC coefficients for a segment consist of these bits, concatenated together. Otherwise \( N>1 \) and the quantized DC coefficients in a segment, \( c'_m \), shall be encoded using the procedure described below.

4.3.2.3 The first quantized DC coefficient for every sequence of \( S \) consecutive coefficients, referred to as a reference sample, shall be written to the encoded bitstream directly (i.e., without any further processing or encoding).

4.3.2.4 For the remaining \( S-1 \) DC coefficients, the difference between successive quantized coefficient values (taken in raster scan order) shall be encoded. Each difference value \( \delta'_m \) in equation 18,

\[
\delta'_m = c'_m - c'_{m-1}
\]  

shall be mapped to a non-negative integer \( \delta_m \) according to the rules in equation 19:

\[
\begin{align*}
\delta_m &= 2(\delta'_m), & \text{if } 0 \leq \delta'_m \leq \theta_m \\
\delta_m &= 2|\delta'_m|-1, & \text{if } -\theta_m \leq \delta'_m < 0 \\
\delta_m &= \theta_m + |\delta'_m|, & \text{otherwise}
\end{align*}
\]

where \( \theta_m = \min(c'_m - x_{\min}, x_{\max} - c'_{m-1}) \), and \( x_{\min} = -2^{N-1}, x_{\max} = 2^{N-1} - 1 \).

4.3.2.5 Each gaggle contains up to 16 mapped quantized coefficients \( \delta_m \):

a) the first gaggle contains 15 values of \( \delta_m \) (the first quantized DC coefficient is coded directly as a reference sample);

b) the remaining gaggles each contain 16 values of \( \delta_m \), with the possible exception of the last gaggle, which may have fewer;

c) if \( S \) is not a multiple of 16, then the last gaggle contains \( J \) values of \( \delta_m \), where \( J \) equals \( S \mod 16 \).

4.3.2.6 The values of \( \delta_m \) in each gaggle shall be encoded using one of several code options ranging from uncoded (that is, each \( \delta_m \) is encoded using the conventional \( N \)-bit unsigned binary integer representation) to encoded via one of several variable-length codes parameterized by a nonnegative integer \( k \).

NOTE – This procedure of prediction, mapping to nonnegative values, and coding follows the CCSDS Lossless Compression standard (reference [B2]) with a slightly simplified entropy coder and different sequences to identify the code option selected for each gaggle.
4.3.2.7 The code option selected for the gaggle shall be indicated at the start of the coded quantized DC coefficients in the gaggle using the appropriate code option identifier selected from table 4-9.

<table>
<thead>
<tr>
<th>Code Parameter $k$</th>
<th>$N=2$</th>
<th>$2 &lt; N \leq 4$</th>
<th>$4 &lt; N \leq 8$</th>
<th>$8 &lt; N \leq 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 0$</td>
<td>0</td>
<td>00</td>
<td>000</td>
<td>0000</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>–</td>
<td>01</td>
<td>001</td>
<td>0001</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>–</td>
<td>10</td>
<td>010</td>
<td>0010</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>–</td>
<td>–</td>
<td>011</td>
<td>0011</td>
</tr>
<tr>
<td>$k = 4$</td>
<td>–</td>
<td>–</td>
<td>100</td>
<td>0100</td>
</tr>
<tr>
<td>$k = 5$</td>
<td>–</td>
<td>–</td>
<td>101</td>
<td>0101</td>
</tr>
<tr>
<td>$k = 6$</td>
<td>–</td>
<td>–</td>
<td>110</td>
<td>0110</td>
</tr>
<tr>
<td>$k = 7$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0111</td>
</tr>
<tr>
<td>$k = 8$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>1000</td>
</tr>
<tr>
<td>uncoded</td>
<td>1</td>
<td>11</td>
<td>111</td>
<td>1111</td>
</tr>
</tbody>
</table>

NOTE: ‘–’ indicates no applicable value.

4.3.2.8 When the uncoded option is selected, the coded gaggle is fixed in length, consisting of the option ID field, optionally followed by an $N$-bit reference sample, and the $J$ values of $\delta_m$ as shown in figure 4-8.

4.3.2.9 Otherwise, given code parameter $k$, the variable-length codeword for $\delta_m$ has two parts. The first part shall consist of $z$ zeros followed by a 1, where $z = \lfloor \delta_m/2^k \rfloor$. The second part shall consist of the $k$ least-significant bits of the binary representation of $\delta_m$.

NOTE – $z$ can be computed by performing $k$ right bit-shifts of the binary representation of $\delta_m$.

4.3.2.10 The coded values of $\delta_m$ within a gaggle shall consist of the code option identifier, then a reference sample (for the first gaggle in a segment), next the corresponding sequence of first part codewords for the samples in a gaggle, followed by the second part codewords as shown in figure 4-9.
a) first gaggle in a segment:

<table>
<thead>
<tr>
<th>code option $k$</th>
<th>$N$-bit reference</th>
<th>15 mapped sample differences, $\delta_m$</th>
</tr>
</thead>
</table>

(fixed-length)

b) subsequent gaggles:

<table>
<thead>
<tr>
<th>code option $k$</th>
<th>$J$ mapped sample differences, $\delta_m$</th>
</tr>
</thead>
</table>

(fixed-length)

Figure 4-8: Coded Data Format for a Gaggle When Uncoded Option Is Selected

a) first gaggle in a segment:

<table>
<thead>
<tr>
<th>code option $k$</th>
<th>$N$-bit reference</th>
<th>first part words, (15 samples)</th>
<th>second part words, ($k\cdot15$ bits)</th>
</tr>
</thead>
</table>

(variable-length)

b) subsequent gaggles:

<table>
<thead>
<tr>
<th>code option $k$</th>
<th>first part words, ($J$ samples)</th>
<th>second part words, ($k\cdot J$ bits)</th>
</tr>
</thead>
</table>

(variable-length)

Figure 4-9: Coded Data Format for a Gaggle When a Coding Option Is Selected

4.3.2.11 For a given application, a user shall use one of the following methods for selecting the code option:

a) select the optimum value of $k$ for each gaggle (i.e., select the value of $k$ that minimizes the number of encoded bits;

NOTE – This optimization could be performed by explicitly computing the number of encoded bits under each allowed value of $k$.

b) select the value of $k$ using the following simplified (but in general suboptimal) heuristic procedure:
Let \( \Delta \) denote the sum of the \( \delta_m \) values in a gaggle:

\[
\Delta = \sum_m \delta_m
\]

The code option is selected depending on the value of the sum \( \Delta \) according to rules in table 4-10.

**Table 4-10: Code Option Selection Rules**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Code Option Selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 64 \cdot \Delta \geq 23 \cdot J \cdot 2^N )</td>
<td>uncoded</td>
</tr>
<tr>
<td>( 207 \cdot J &gt; 128 \cdot \Delta )</td>
<td>( k=0 )</td>
</tr>
<tr>
<td>( J \cdot 2^{k+7} \leq 128 \cdot \Delta + 49 \cdot J )</td>
<td>( k=N-2 )</td>
</tr>
<tr>
<td>otherwise</td>
<td>( k ) is the largest nonnegative integer less than or equal to ( N-2 ) such that ( J \cdot 2^{k+7} \leq 128 \cdot \Delta + 49 \cdot J )</td>
</tr>
</tbody>
</table>

NOTE – Other methods of selecting the parameter \( k \) are not considered compliant with this Recommended Standard.

4.3.2.12 The method used to select the parameter \( k \) shall be indicated in Segment Header Part 3, when Segment Header Part 3 header is included.

NOTE – Since the value of the parameter \( k \) is explicitly encoded in the compressed data stream, it is not necessary for the decoder to know whether the optimum or the heuristic selection method was used.

4.3.2.13 When the optimum code option selection method is used, the following rules shall be applied when the optimum code option is not unique:

a) The uncoded option shall be selected whenever it minimizes the number of encoded bits, even if another option gives the same number of bits.

b) When two or more code parameters minimize the number of encoded bits, the smallest code parameter option shall be selected.

4.3.3 ADDITIONAL BIT PLANES OF DC COEFFICIENTS

NOTE – The coding of quantized DC coefficients described in 4.3.2 effectively encodes the first BitDepthDC-\( q \) bits of each DC coefficient.

4.3.3.1 When \( q \) >BitDepthAC, the next \( q \)-BitDepthAC bits of each DC coefficient shall appear in the coded bitstream immediately following the coded quantized DC coefficients of the segment (described in 4.3.2).
4.3.3.2 The appropriate bits of each DC coefficient in a segment shall be concatenated for each bit plane; that is, the \((q-1)^{th}\) most-significant bit of each DC coefficient shall be followed by the \((q-2)^{th}\) most-significant bit of each DC coefficient, all uncoded, and so on, until the BitDepthAC\(^{th}\) bit of each DC coefficient.

4.4 SPECIFYING THE AC BIT DEPTH IN EACH BLOCK

The first step in encoding AC coefficient magnitudes in a segment is to specify the sequence of BitDepthAC\(_m\) values for the segment. These values shall be coded in one of three ways, depending on the value of BitDepthAC that is encoded in Part 1A of the segment header:

a) If BitDepthAC is zero, then the sequence of BitDepthAC\(_m\) values are zero and thus shall not be coded. In addition, no bit plane coding of the AC coefficients shall be performed since all of the AC coefficients in the segment must be zero.

b) If BitDepthAC is one, then each BitDepthAC\(_m\) value is either zero or one, and thus each BitDepthAC\(_m\) value shall be encoded using a single bit to indicate its value; i.e., the single-bit BitDepthAC\(_m\) values shall be concatenated.

c) Otherwise, the sequence of BitDepthAC\(_m\) values for the segment shall be coded using the same differencing and variable-length coding procedure specified for coding quantized DC values described in 4.3.2. However, a few parameters differ for the case of coding the BitDepthAC\(_m\) values:

1) \(N\) shall equal the number of bits required to represent the magnitude of BitDepthAC for the segment, as in equation 21:

\[
N = \lceil \log_2(1 + \text{BitDepthAC}) \rceil \tag{21}
\]

For code option identification, table 4-9 shall be used with this value of \(N\).

NOTE – For a 16-bit source image, no more than five bits are needed to specify the value of BitDepthAC, i.e., \(N \leq 5\).

2) Since the BitDepthAC\(_m\) values are necessarily nonnegative, \(x_{\text{min}}\) and \(x_{\text{max}}\) shall be redefined as given in equation 22 and used to map successive differences to nonnegative integers:

\[
x_{\text{min}} = 0, \quad x_{\text{max}} = 2^N - 1 \tag{22}
\]

NOTE – The coded bit string for the BitDepthAC\(_m\) values follows the same data format illustrated in figures 4-8 and 4-9.
4.5 BIT PLANE CODING

4.5.1 OVERVIEW

Coding of a bit plane is performed in stages numbered 0-4. The coded bits produced at the stages for each block are interleaved, as illustrated in figure 4-2(b) and figure 4-10. Thus, a coded bit plane first consists of all the stage 0 bits (if any) in the segment, then all of the coded stage 1 bits in the segment, and so on, finishing with all of the encoded stage 4 bits in the segment. This produces an embedded bit string with information from the highest bit plane of all S blocks in the first part of the output bit string followed by information from lower bit planes, and allows progressive decoding of the coded string. This improves image reconstruction quality when the coded bit sequence is truncated.

<table>
<thead>
<tr>
<th>Stage 0 bits from each block in the segment (if any)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coded stage 1 from block 0, 1, ..., S-1</td>
</tr>
<tr>
<td>Coded stage 2 from block 0, 1, ..., S-1</td>
</tr>
<tr>
<td>Coded stage 3 from block 0, 1, ..., S-1</td>
</tr>
<tr>
<td>Coded stage 4 from block 0, 1, ..., S-1</td>
</tr>
</tbody>
</table>

Figure 4-10: Coded Bit Plane Structure for a Segment

Note that when the index b of the bit plane being coded is larger than or equal to the AC bit depth of the block, then there is nothing to code for the block.

The layered coding stages in figure 4-10 inherently allow a controlled amount of DWT coefficient information to be encoded within a bit plane, permitting image quality control at sub-bit plane levels. The tradeoff between reconstructed image quality and compressed data volume for each segment is controlled by specifying the first four parameters in Part 2 Header (table 4-5). Quality level and data volume are both specified for each segment, and compressed output for a segment is produced until either the quality limit or the volume limit is reached. Users may choose to achieve fixed output rate by using fill bits. These features are all described in 4.2.3.

Annex C contains a list of symbols used in the various coding stages specified in 4.5.2, 4.5.3, and 4.5.4.

4.5.2 OVERVIEW OF CODING STAGES

The coding stages for a block at bit plane b are described in the following paragraphs.

Stage 0 for a block consists of at most a single bit, which is simply the b\textsuperscript{th} most significant bit of the two’s-complement representation of the DC coefficient. Note that whenever the bit plane index b satisfies b ≥ q, this bit value is already known from the DC coefficient information already encoded, and in this case, stage 0 is empty, i.e., no bits are coded in
stage 0. Stage 0 is also empty when scaling of the DC coefficient assures that the bit must be zero, i.e., when \( b < \text{BitShift}(\text{LL}_3) \).

The remaining stages (1-4) encode AC coefficient bits. The stage in which bits from AC coefficients in a bit plane are coded depends on the type of the AC coefficient at the bit plane, which we now define. At bit plane \( b \), the type of an AC coefficient \( x \), denoted \( t_b(x) \), has one of the following values:

- \( t_b(x) = 0 \) if \( |x| < 2^b \), (\( x \) is not due for selection at this bit plane);
- \( t_b(x) = 1 \) if \( 2^b \leq |x| < 2^{b+1} \), (\( x \) is due for selection at this bit plane);
- \( t_b(x) = 2 \) if \( 2^{b+1} \leq |x| \), (\( x \) has already been selected at a previous bit plane);
- \( t_b(x) = -1 \) if \( b < \text{BitShift}(\Gamma) \), (\( x \) must be zero at this bit plane due to subband scaling).

Here, \( \Gamma \) denotes the subband containing \( x \). Thus, during bit-plane encoding, each AC coefficient typically proceeds from type 0 to 1, to 2, to -1. For a set of coefficients \( \Psi \), we define \( t_{\max}(\Psi) \) as the maximum of the coefficient types in \( \Psi \).

An AC coefficient \( x \) is said to be selected at bit plane \( b \) if \( t_b(x) = 1 \). I.e., the ‘selection’ of a coefficient marks the first bit plane where a non-zero magnitude bit is encoded for the coefficient. Note that \( t_b(x) = 1 \) if the \( b \)th most significant magnitude bit of \( x \) is equal to ‘1’ and all more significant magnitude bits of \( x \) are equal to ‘0’.

The type of a coefficient determines the stage when coding of a coefficient bit takes place. When an AC coefficient \( x \) is of type 0 or 1 (implying \( t_{b+1}(x) = 0 \)), the \( b \)th most significant magnitude bit of \( x \) is coded in stages 1-3. Otherwise, the bit is included, uncompressed, in stage 4 if \( x \) is of type 2, or not encoded at all when \( x \) is of type -1.

In stages 1-3 of BPE encoding bit plane \( b \), the \( b \)th magnitude bit of each AC coefficient \( x \) such that \( t_{b+1}(x) = 0 \) is encoded. The \( b \)th magnitude bits of the parent coefficients are coded in stage 1, the children in stage 2, and the grandchildren in stage 3. Each of these stages also includes coded bits indicating the sign of each coefficient \( x \) for which \( t_b(x) = 1 \). The coding in stages 1-3 makes use of the family structure to group together AC coefficients for entropy coding.

The coding performed in stages 1-3 for a block consists of two parts. First, a sequence of variable length binary words are defined which completely describe the bits to be encoded in these stages, as discussed in 4.5.3.1. Next, a subset of these words are further entropy coded, as described in 4.5.3.3.

Stage 4 of coding consists of the \( b \)th magnitude bit of each AC coefficient \( x \) with \( t_b(x) = 2 \). These bits are included in the coded data stream uncompressed, in the order specified in 4.5.4.
4.5.3 CODING STAGES 1-3

4.5.3.1 AC Coefficient Words

NOTE – The bits encoded in stages 1-3 for a block can be determined by a sequence of words, as described below. Some of these words are further entropy coded, as described in 4.5.3.3.

4.5.3.1.1 In addition to the sets $C_i$, $G_i$, $H_{ij}$, defined in 4.1, $P$ is defined as the list of parents in the block:

$$P = \{p_0, p_1, p_2\}.$$

4.5.3.1.2 The list of descendants in family $i$, denoted $D_i$, is defined as

$$D_i = \{C_i, G_i\}.$$

4.5.3.1.3 The list of descendants in a block, denoted $B$, is defined as

$$B = \{D_0, D_1, D_2\}.$$

NOTE – $\{A, B\}$ denotes the concatenation of the lists $A$ and $B$.

4.5.3.1.4 A shorthand notation for certain binary words that describe information about bit plane $b$ for a list of coefficients $\Psi$ is defined as follows:

- let $\text{types}_b[\Psi]$ denote the binary word consisting of the $b^{\text{th}}$ magnitude bit of each coefficient $x$ in $\Psi$ such that $t_b(x)$ equals 0 or 1;
- let $\text{signs}_b(\Psi)$ denote the binary word consisting of the sign bit of each coefficient $x$ in $\Psi$ such that $t_b(x) = 1$, with a sign bit of ‘1’ for negative coefficients and ‘0’ for nonnegative coefficients;
- given a list of type values $\Lambda=\{\lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_l\}$, let $\text{tword}[\Lambda]$ denote the binary word consisting of the sequence of type values $\lambda_i$ in $\Lambda$ that are equal to 0 or 1.

NOTE – Any of these words can be null (i.e., have length zero).

4.5.3.1.5 The list $P$ shall be ordered $P = \{p_0, p_1, p_2\}$, while the ordering on the lists $C_i$ and $H_{ij}$ shall be determined by the order in which their member coefficients’ coordinates are listed in table 4-1.

4.5.3.1.6 The $b^{\text{th}}$ magnitude bits for all AC coefficients that are type 0 at bit plane $b+1$ (i.e., not selected before the current bit plane) shall be communicated to the decoder by joining them to form binary words associated with each data type (parent, child, grandchild):

- $\text{types}_b[P]$;
- $\text{types}_b[C_i]$ for $i = 0, 1, 2$; and
- $\text{types}_b[H_{ij}]$ for $i = 0, 1, 2$, $j = 0, 1, 2, 3$. 
4.5.3.1.7 At early bit planes, many sets of coefficients in a block tend to all be of type 0, and thus many of these words are initially all zeros. To effectively encode in this situation, the BPE shall make use of the following transition words to indicate when groups of coefficients at a lower depth are all Type 0:

- \( \text{tran}_B = \begin{cases} \text{null,} & \text{if } \text{tran}_B = 1 \text{ at any more significant bit plane,} \\ \text{tword}\{t_{max}(B)\}, & \text{otherwise;} \end{cases} \)

- \( \text{tran}_D = \text{tword}\{t_{max}(D_i) : i=0,1,2, \text{ such that } t_{max}(D_i) \neq 1 \text{ in all more significant bit planes}\} \);

- \( \text{tran}_G = \text{tword}\{t_{max}(G_i) : i=0,1,2, \text{ such that } t_{max}(D_i) > 0 \text{ in current or any more significant bit planes}\} \);

- \( \text{tran}_{Hi} = \text{tword}\{t_{max}(H_{i0}), t_{max}(H_{i1}), t_{max}(H_{i2}), t_{max}(H_{i3})\} \text{ for } i = 0,1,2. \)

4.5.3.1.8 At bit plane \( b \), the BPE shall use the following sequence of words, generated in three stages, to update all of the AC coefficients in the block that were Type 0 at the previous bit plane:

a) **Stage 1 (parents):**

\( \text{types}_b[P], \text{signs}_b[P] \).

b) **Stage 2 (children):**

1) \( \text{tran}_B \);

2) \( \text{tran}_D \), if \( \text{tran}_B \neq 0 \) and \( t_{max}(B) \neq -1 \);

3) \( \text{types}_b[C_i] \) and \( \text{signs}_b(C_i) \) for each \( i \) such that \( t_{max}(D_i) \neq 0, -1 \).

c) **Stage 3 (grandchildren):**

If \( \text{tran}_B = 0 \) or \( t_{max}(B) = -1 \), then stage 3 is unnecessary and shall be omitted. Otherwise stage 3 consists of:

1) \( \text{tran}_G \);

2) \( \text{tran}_{Hi} \), for each \( i \) such that \( t_{max}(G_i) \neq 0, -1 \);

3) \( \text{types}_b[H_{ij}] \) and \( \text{signs}_b[H_{ij}] \) for each \( i \) such that \( t_{max}(G_i) \neq 0, -1 \) and each \( j \) such that \( t_{max}(H_{ij}) \neq 0, -1 \).

**NOTE** – All of the words generated in the above stages are variable length (including the null word).

4.5.3.1.9 Words \( \text{types}_b[P], \text{types}_b[C_i], \text{types}_b[H_{ij}], \text{tran}_D, \text{tran}_G, \text{tran}_{Hi} \) shall be entropy coded, i.e., each shall be replaced with a corresponding variable-length codeword, whenever such a word has a length of at least 2 bits.
NOTE – The sign bit words are not coded further, because AC coefficients are generally positive and negative with about equal probability. The tran\textsubscript{B} word is always, at most, one bit in length and is never entropy coded.

4.5.3.2 Mapping Words to Symbols

4.5.3.2.1 The entropy coding procedure used to encode the words \textit{types\textsubscript{b}}[P], \textit{types\textsubscript{b}}[Ci], \textit{types\textsubscript{b}}[Hij], tran\textsubscript{D}, tran\textsubscript{G}, tran\textsubscript{Hi} shall be accomplished through the use of variable-length codes given in 4.5.3.3. Words having a length of one bit, and sign-bit words, shall be included in the compressed data stream without further coding. Words of length greater than one bit shall be coded in the sequence in which they occur within each stage with the entropy coding method described in 4.5.3.3.

NOTE – Certain bit sequences cannot appear as values for certain words and this fact is taken into account in the entropy coding process. For example, tran\textsubscript{D} can never equal 000, because this condition would be inferred from the fact that tran\textsubscript{B}=0. Table 4-11 summarizes the maximum word lengths and impossible values for each word that is entropy coded.

Table 4-11: Summary of Maximum Word Lengths and Impossible Word Values

<table>
<thead>
<tr>
<th>Word</th>
<th>Maximum Length (bits)</th>
<th>Impossible Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>types\textsubscript{b}[P]</td>
<td>3</td>
<td>–</td>
</tr>
<tr>
<td>types\textsubscript{b}[Ci]</td>
<td>4</td>
<td>–</td>
</tr>
<tr>
<td>types\textsubscript{b}[Hij]</td>
<td>4</td>
<td>0000</td>
</tr>
<tr>
<td>tran\textsubscript{D}</td>
<td>3</td>
<td>000</td>
</tr>
<tr>
<td>tran\textsubscript{G}</td>
<td>3</td>
<td>–</td>
</tr>
<tr>
<td>tran\textsubscript{Hi}</td>
<td>4</td>
<td>0000</td>
</tr>
</tbody>
</table>

4.5.3.2.2 The process of variable-length coding of these words shall follow a two-step process:

– first, the word values shall be mapped to integer values referred to as symbols; then
– each integer shall be encoded using a variable-length binary codeword described in 4.5.3.3.

4.5.3.2.3 Under the mapping, two-bit, three-bit, and four-bit words shall be mapped to symbols using table 4-12, 4-13, or 4-14, respectively.

NOTE – The mapping process takes into account the fact that certain words can never be assigned certain bit sequences, as tabulated in table 4-11; this is reflected in tables 4-13 and 4-14.
Table 4-12: Integer Mapping for Two-Bit Words

<table>
<thead>
<tr>
<th>Word</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>0</td>
</tr>
<tr>
<td>01</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4-13: Integer Mapping for Three-Bit Words

<table>
<thead>
<tr>
<th>Word</th>
<th>Symbol (typesb[P], typesb[C], typesb[H], tranG, tranH)</th>
<th>Symbol (tranD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>001</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>011</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>101</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>110</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>111</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4-14: Integer Mapping for Four-Bit Words

<table>
<thead>
<tr>
<th>Word</th>
<th>Symbol (typesb[C])</th>
<th>Symbol (typesb[H], tranH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0010</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0011</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>0100</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0101</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>0110</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>0111</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1001</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>1010</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>1011</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>1100</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>1101</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>1110</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>1111</td>
<td>15</td>
<td>14</td>
</tr>
</tbody>
</table>
NOTE – The mappings are intended to produce symbol values in order of decreasing frequency. (I.e., the most frequently occurring word is mapped to symbol value 0, the next most frequent to 1, etc.) This makes effective coding possible through the coding procedure described in 4.5.3.3 because the codewords are arranged in order of increasing length.

4.5.3.3 Entropy Coding the Symbols

4.5.3.3.1 The symbols of 4.5.3.2 shall be encoded using the variable-length binary codes given in tables 4-15, 4-16, and 4-17.

4.5.3.3.2 Within a gaggle,

- all two-bit words shall be encoded using one of the two variable-length code options given in table 4-15;
- all three-bit words shall be encoded using one of the three variable-length code options given in table 4-16;
- all four-bit words shall be encoded using one of the four variable-length code options given in table 4-17.

4.5.3.3.3 The variable-length codes used are permitted to change with each gaggle.

Table 4-15: Variable Length Code Options for Two-Bit Words

<table>
<thead>
<tr>
<th>Input Symbol</th>
<th>Code Option 0</th>
<th>Code Option 1 (uncoded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>00</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
<td>01</td>
</tr>
<tr>
<td>2</td>
<td>001</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>000</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 4-16: Variable Length Code Options for Three-Bit Words

<table>
<thead>
<tr>
<th>Input Symbol</th>
<th>Code Option 0</th>
<th>Code Option 1</th>
<th>Code Option 3 (uncoded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>10</td>
<td>000</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
<td>11</td>
<td>001</td>
</tr>
<tr>
<td>2</td>
<td>001</td>
<td>010</td>
<td>010</td>
</tr>
<tr>
<td>3</td>
<td>0000</td>
<td>011</td>
<td>011</td>
</tr>
<tr>
<td>4</td>
<td>00001</td>
<td>0010</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>00010</td>
<td>0011</td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>000110</td>
<td>000</td>
<td>110</td>
</tr>
<tr>
<td>7</td>
<td>000111</td>
<td>0001</td>
<td>111</td>
</tr>
</tbody>
</table>
Table 4-17: Variable Length Code Options for Four-Bit Words

<table>
<thead>
<tr>
<th>Input Symbol</th>
<th>Code Option 0</th>
<th>Code Option 1</th>
<th>Code Option 2</th>
<th>Code Option 3 (uncoded)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>01</td>
<td>10</td>
<td>100</td>
<td>0000</td>
</tr>
<tr>
<td>1</td>
<td>101</td>
<td>11</td>
<td>101</td>
<td>0001</td>
</tr>
<tr>
<td>2</td>
<td>001</td>
<td>010</td>
<td>110</td>
<td>0010</td>
</tr>
<tr>
<td>3</td>
<td>0001</td>
<td>011</td>
<td>111</td>
<td>0011</td>
</tr>
<tr>
<td>4</td>
<td>0000000</td>
<td>0010</td>
<td>0100</td>
<td>0100</td>
</tr>
<tr>
<td>5</td>
<td>0000001</td>
<td>0011</td>
<td>0101</td>
<td>0101</td>
</tr>
<tr>
<td>6</td>
<td>0000010</td>
<td>000000</td>
<td>0110</td>
<td>0110</td>
</tr>
<tr>
<td>7</td>
<td>0000011</td>
<td>000001</td>
<td>0111</td>
<td>0111</td>
</tr>
<tr>
<td>8</td>
<td>00001000</td>
<td>000010</td>
<td>00100</td>
<td>1000</td>
</tr>
<tr>
<td>9</td>
<td>00001001</td>
<td>000011</td>
<td>0101</td>
<td>1001</td>
</tr>
<tr>
<td>10</td>
<td>00001010</td>
<td>000100</td>
<td>0110</td>
<td>1010</td>
</tr>
<tr>
<td>11</td>
<td>00001011</td>
<td>000101</td>
<td>0111</td>
<td>1011</td>
</tr>
<tr>
<td>12</td>
<td>00001100</td>
<td>000110</td>
<td>0000</td>
<td>1100</td>
</tr>
<tr>
<td>13</td>
<td>00001101</td>
<td>0001101</td>
<td>0001</td>
<td>1101</td>
</tr>
<tr>
<td>14</td>
<td>00001110</td>
<td>0001110</td>
<td>00010</td>
<td>1110</td>
</tr>
<tr>
<td>15</td>
<td>00001111</td>
<td>0001111</td>
<td>00011</td>
<td>1111</td>
</tr>
</tbody>
</table>

4.5.3.3.4 For each word size within a gaggle (i.e., two-bit, three-bit, and four-bit words), the code option selected shall be the one that minimizes the encoded length of the gaggle when all of the words are coded. The code parameter optimizations are performed over all of the words in the gaggle even when the quality output mode specified ensures that some of those words are not included in the compressed bit stream. The code option selected shall be indicated using ID bits that take on the values given in table 4-18.

NOTE – At most five bits per gaggle per bit plane are needed to specify the code options.

4.5.3.3.5 The uncoded option shall be selected whenever it minimizes the number of encoded bits, even if another option gives the same number of bits. When two or more code parameters minimize the number of encoded bits, the smallest code parameter option shall be selected.
Table 4-18: Identifying the Variable Length Code Options

<table>
<thead>
<tr>
<th>No. of Bits for Mapped Patterns</th>
<th>No. of ID Bits</th>
<th>ID and Associated Code Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0: code option 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1: uncoded</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>00: code option 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>01: code option 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11: uncoded</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>00: code option 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>01: code option 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10: code option 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11: uncoded</td>
</tr>
</tbody>
</table>

4.5.3.3.6 The ID bits for specifying coding options for words of a given length shall be inserted immediately preceding the first appearance of a codeword for a given length within a gaggle. When no word of the given length occurs in a gaggle, no ID bits shall be present.

4.5.3.3.7 The coded word shall replace the appropriate uncoded word identified in 4.5.3.1.

4.5.4 STAGE 4 CODING

4.5.4.1 In stage 4 of coding, the $b^{th}$ magnitude bit of each AC coefficient $x$ with type $t_b(x)=2$ shall be included in the output bit stream.

4.5.4.2 For each block, the output bit string shall consist of the $b^{th}$ magnitude bit of type 2 coefficients, in the following order:
   - $p_i$, for each $i = 0,1,2$;
   - members of $C_i$, for each $i = 0,1,2$;
   - members of $H_{ij}$, for each $i = 0,1,2$, and each $j = 0,1,2,3$.

4.5.4.3 Members of the sets $C_i$ and $H_{ij}$ shall be processed in the order listed in table 4-1. No bits shall be coded in stage 4 for AC coefficients $x$ not of type 2 ($t_b(x) \neq 2$).

4.5.4.4 The resulting strings for all blocks in the segment shall be concatenated to produce the entire stage 4 output string for the segment.
5 SECURITY

5.1 INTRODUCTION

This section presents the results of an analysis of security considerations applied to the technologies specified in this Recommended Standard.

5.2 SECURITY CONCERNS WITH RESPECT TO THIS RECOMMENDED STANDARD

5.2.1 DATA PRIVACY

Privacy of data compressed in compliance with the specifications of this Recommended Standard should be assured by the systems and networks on which this Recommended Standard is implemented.

5.2.2 DATA INTEGRITY

Integrity of data compressed in compliance with the specifications of this Recommended Standard should be assured by the systems and networks on which this Recommended Standard is implemented.

5.2.3 AUTHENTICATION OF COMMUNICATING ENTITIES

Authentication of communication entities involved in the transport of data compressed in compliance with the specifications of this Recommended Standard should be provided by the systems and networks on which this Recommended Standard is implemented.

5.2.4 CONTROL OF ACCESS TO RESOURCES

This Recommended Standard assumes that control of access to resources will be managed by the systems on which compressor and decompressor entities reside.

5.2.5 AVAILABILITY OF RESOURCES

This Recommended Standard assumes an adequate availability of resources on the systems on which compressor and decompressor entities reside.

5.2.6 AUDITING OF RESOURCE USAGE

This Recommended Standard assumes that auditing of resource usage will be handled by the management of systems and networks on which this Recommended Standard is implemented.
5.3 POTENTIAL THREATS AND ATTACK SCENARIOS

There are no known potential threats or attack scenarios that apply specifically to the technologies specified in this Recommended Standard. Potential threats or attack scenarios applicable to the systems and networks on which this Recommended standard is implemented should be addressed by the management of those systems and networks.

5.4 CONSEQUENCES OF NOT APPLYING SECURITY TO THE TECHNOLOGY

There are no known consequences of not applying security to the technologies specified in this Recommended Standard. The consequences of not applying security to the systems and networks on which this Recommended Standard is implemented include potential loss, corruption, and theft of data.
## ANNEX A

### GLOSSARY OF ACRONYMS AND TERMS

*(Informative)*

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BPE</strong></td>
<td>Bit-plane encoder: recommended processing algorithm used to compress wavelet coefficient data.</td>
</tr>
<tr>
<td><strong>DWT</strong></td>
<td>Discrete Wavelet Transform: recommended processing algorithm to transform image data to wavelet coefficient data.</td>
</tr>
<tr>
<td><strong>CCD</strong></td>
<td>Charge Coupled Device, an imaging sensor.</td>
</tr>
<tr>
<td><strong>9/7 DWT</strong></td>
<td>A DWT using a 9-tap filter to obtain low-pass wavelet coefficients and a 7-tap filter to obtain high-pass wavelet coefficients. Two different specific 9/7 Discrete Wavelet Transforms are recommended: 9/7 Float DWT for lossy compression and 9/7 Integer DWT for lossless compression.</td>
</tr>
<tr>
<td><strong>9/7 Float DWT</strong></td>
<td>Three-level 9/7 DWT whose filter coefficients are specified real numbers. Implemented using float arithmetic. Cf. equation (3) and table 3-1.</td>
</tr>
<tr>
<td><strong>9/7 Integer DWT</strong></td>
<td>Three-level 9/7 DWT whose filter coefficients are specified rational numbers. Implemented using integer arithmetic. Cf. equations (5) and (6).</td>
</tr>
<tr>
<td><strong>MSB</strong></td>
<td>Most Significant Bit (of a word): left-most bit in figures: highest power of two in binary representation; first bit to be transmitted in serial output.</td>
</tr>
<tr>
<td><strong>LL&lt;sub&gt;n&lt;/sub&gt;</strong></td>
<td>2-d subband at n-th level of DWT; for n=1 obtained after sequentially applying low-pass 1-d DWT filter to horizontal and vertical lines of image; for n&gt;1 obtained after sequentially applying low-pass 1-d DWT filter to horizontal and vertical lines of subband LL&lt;sub&gt;n-1&lt;/sub&gt;.</td>
</tr>
<tr>
<td><strong>LH&lt;sub&gt;n&lt;/sub&gt;</strong></td>
<td>2-d subband at n-th level of DWT; for n=1 obtained after sequentially applying low-pass 1-d DWT filter to horizontal, and high-pass 1-d DWT filter to vertical lines of image; for n&gt;1 obtained after sequentially applying low-pass 1-d DWT filter to horizontal, and high-pass 1-d DWT filter to vertical lines of subband LL&lt;sub&gt;n-1&lt;/sub&gt;.</td>
</tr>
</tbody>
</table>
HL<sub>n</sub> 2-d subband at <i>n</i>-th level of DWT; for <i>n</i>=1 obtained after sequentially applying high-pass 1-d DWT filter to horizontal, and low-pass 1-d DWT filter to vertical lines of image; for <i>n</i>&gt;1 obtained after sequentially applying high-pass 1-d DWT filter to horizontal, and low-pass 1-d DWT filter to vertical lines of subband LL<sub><i>n</i>-1</sub>.

HH<sub>n</sub> 2-d subband at <i>n</i>-th level of DWT; for <i>n</i>=1 obtained after sequentially applying high-pass 1-d DWT filter to horizontal and vertical lines of image; for <i>n</i>&gt;1 obtained after sequentially applying high-pass 1-d DWT filter to horizontal and vertical lines of subband LL<sub><i>n</i>-1</sub>.

weight factors after transforming image data by means of the 9/7 integer DWT, the obtained wavelet coefficients need to be multiplied by these numbers before encoding with the BPE. One weight factor is defined for each subband.

DC coefficient any wavelet coefficient from the lowest frequency subband LL<sub>3</sub>.

AC coefficient any wavelet coefficient from any subband except LL<sub>3</sub>.

block Collection of 64 wavelet coefficients, consisting of one DC coefficient from the LL<sub>3</sub> subband and 63 uniquely associated AC coefficients from the remaining nine subbands (see figure 4-1 and table 4-1). For encoding, the complete wavelet domain is divided into blocks that are pairwise disjoint. There is one block for each DC coefficient.

segment Bitstream of compressed code consisting of a data field headed by a segment header. The segment header is defined in 4.2. The data field contains the encoded bits from <i>S</i> consecutive blocks. <i>S</i> is a user-selected parameter such that 16≤<i>S</i>≤220.

gaggle A gaggle consists of a set of consecutive blocks within a segment. Specifically, a segment is partitioned into gaggles, with each gaggle consisting of 16 blocks, except possibly the last gaggle, which contains <i>(S</i> mod 16) blocks when <i>S</i> is not a multiple of 16 (see 4.3.2.5). Certain code parameters are independently selected for each gaggle. Specifically, all of the quantized DC coefficient values in a gaggle are coded using the same code option (see 4.3.2.6). Similarly, at each bit plane, the same set of variable-length code options are used for all encoded variable length words in a gaggle (see 4.5.3.3.3).
ANNEX B

INFORMATIVE REFERENCES

(Informative)


## ANNEX C

### SYMBOLS USED IN CODING STAGES

*(Informative)*

Table C-1: Symbols Used in Coding Stages in Section 4

<table>
<thead>
<tr>
<th>Category</th>
<th>Symbols</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>wavelet coefficients</td>
<td>$DC, c$</td>
<td>DC coefficient</td>
</tr>
<tr>
<td></td>
<td>$c'$</td>
<td>quantized DC coefficient</td>
</tr>
<tr>
<td></td>
<td>$AC, x$</td>
<td>AC coefficient</td>
</tr>
<tr>
<td></td>
<td>$p_i$</td>
<td>parent coefficient ($i=0,1,2$)</td>
</tr>
<tr>
<td>ordered sets of wavelet coefficients assigned to a block</td>
<td>$\Psi$</td>
<td>variable set</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
<td>set of all descendent coefficients</td>
</tr>
<tr>
<td></td>
<td>$P$</td>
<td>set of parent coefficients</td>
</tr>
<tr>
<td></td>
<td>$D_i$</td>
<td>set of descendent coefficients ($i=0,1,2$)</td>
</tr>
<tr>
<td></td>
<td>$C_i$</td>
<td>set of children coefficients ($i=0,1,2$)</td>
</tr>
<tr>
<td></td>
<td>$G_i$</td>
<td>set of grandchildren coefficients ($i=0,1,2$)</td>
</tr>
<tr>
<td></td>
<td>$H_{ij}$</td>
<td>subset of set of grandchildren $G_i$ ($i=0,1,2; j=0,1,2,3$)</td>
</tr>
<tr>
<td>integer valued functions</td>
<td>$t_b(x)$</td>
<td>value indicating type of coefficient $x$, with respect to current bit plane with index $b$</td>
</tr>
<tr>
<td></td>
<td>$\max_{\Psi} {t_b(x) : x \in \Psi}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{types}_{\Psi}$</td>
<td>binary word whose bits are the types $t_b(x)$ of the coefficients $x \in \Psi$, ignoring coefficients with types other than $t_b(x)=0$ or $t_b(x)=1$</td>
</tr>
<tr>
<td></td>
<td>$\text{signs}_{\Psi}$</td>
<td>binary word whose bits are the signs of the coefficients $x \in \Psi$, ignoring coefficients whose types are not $t_b(x)=1$. Positive signs give 0, negative signs give 1.</td>
</tr>
<tr>
<td></td>
<td>$\text{tword}([t_0, t_1, \ldots, t_l])$</td>
<td>binary words whose bits are the type values $[t_0, t_1, \ldots, t_l]$, ignoring types whose values are not 0 or 1</td>
</tr>
<tr>
<td>binary words</td>
<td>$\text{tran}_B$</td>
<td>transition word for set $B$</td>
</tr>
<tr>
<td></td>
<td>$\text{tran}_D$</td>
<td>transition word for collection of sets $D_0, D_1, D_2$</td>
</tr>
<tr>
<td></td>
<td>$\text{tran}_G$</td>
<td>transition word for collection of sets $G_0, G_1, G_2$</td>
</tr>
<tr>
<td></td>
<td>$\text{tran}_H_i$</td>
<td>transition word for collection of sets $H_0, H_n$, $H_2, H_3$ ($i=0,1,2$)</td>
</tr>
</tbody>
</table>