Gradient Pattern Analysis: A Preliminary Implementation as a Python Library

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Abstract. *Gradient Pattern Analysis (GPA) is a matrix operator for characterizing the spatial organization of scalar lattices through gradient and symmetry relations. Originally developed to quantify asymmetric fragmentation, GPA has been applied to various physical and astronomical systems. In this work, we present a preliminary Python library that consolidates GPA into a reproducible and extensible framework. The library implements gradient construction, symmetry analysis, gradient moments, and visualization. The current version provides the first two moments, G_1 and G_2, and serves as a foundation for future extensions of the methodology.*

1. Introduction

Quantitative characterization of spatial patterns requires methods capable of representing not only global properties, but also geometric relationships and local asymmetries. Gradient-based methods provide such a representation by transforming lattice variations into vectors associated with grid positions.

Gradient Pattern Analysis (GPA) emerged in this context. Rosa, Sharma, and Valdivia [Rosa et al. 1999] initially proposed a matrix operator to characterize asymmetric fragmentation, in which the gradient lattice is analyzed to identify symmetry relationships and obtain its asymmetric component. Subsequent developments extended the methodology to spatiotemporal patterns and introduced descriptors associated with the geometry, magnitude, and phase of gradients [Rosa et al. 2000, Ramos et al. 2000, Rosa et al. 2003].

The methodology has been applied to different physical systems, materials, cosmological structures, and spatial data, including studies of galaxy morphology and automated classification [Rosa et al. 2000, Ramos et al. 2000, Rosa et al. 2003, Rosa et al. 2008, Sautter 2018, Barchi et al. 2020, Kolesnikov et al. 2024, Kolesnikov et al. 2025, de Souza et al. 2026]. These studies demonstrate that features derived from the organization of gradient lattices can complement traditional morphological descriptors and contribute to classification tasks.

Despite this trajectory, GPA developments remain distributed across different studies and implementations. In this work, we present a preliminary version of a Python library that brings together its main procedures into a single, reusable, and extensible framework, consolidating GPA as an accessible computational tool for feature extraction from spatial patterns.

2. Gradient Pattern Analysis

Gradient Pattern Analysis (GPA) transforms a two-dimensional scalar lattice $M(x, y)$ into a vector field of local gradients,

$$\nabla M(x, y) = (G_x(x, y), G_y(x, y)) . \tag{1}$$

Given a reference center (c_x, c_y), gradient vectors are grouped according to their radial distance from the center. Vectors whose magnitude is below a specified fraction of the maximum gradient magnitude are first excluded. The remaining vectors are then compared within radial-distance groups to identify approximately symmetric pairs. For a gradient vector located at (x_1, y_1), its symmetric counterpart is searched near the position

$$x_2 \simeq 2c_x - x_1, \qquad y_2 \simeq 2c_y - y_1. \tag{2}$$

A candidate pair is considered symmetric when the two vectors have similar magnitudes and approximately opposite orientations,

$$||\mathbf{v}_1| - |\mathbf{v}_2|| \leq \epsilon_m \max |\nabla M|, \qquad |\Delta\theta - \pi| \leq \epsilon_\theta. \tag{3}$$

while their positions are constrained by a specified tolerance around the center-reflected position. Symmetric pairs are then removed, yielding $\nabla M_{\rm asym}$.

These operations follow the GPA framework, in which symmetric contributions are removed to characterize asymmetric organization in spatially extended systems [Rosa et al. 1999, Rosa et al. 2003]. The resulting field defines the gradient moments,

$$M \rightarrow \nabla M \rightarrow \nabla M_{\rm asym} \rightarrow \{G_1, G_2, G_3, G_4\}. \tag{4}$$

In the current implementation, G_1 and G_2 are available, whereas G_3 and G_4 are reserved for future implementations. The first moment, G_1, characterizes the geometric organization of the asymmetric lattice through a Delaunay triangulation,

$$G_1 = \frac{T_{\rm A} - V_{\rm A}}{V_{\rm A}}, \tag{5}$$

where $T_{\rm A}$ is the number of unique connections in the triangulation and $V_{\rm A}$ is the number of non-zero asymmetric vectors [Rosa et al. 1999, Rosa et al. 2003].

The second moment, G_2, combines the fraction of asymmetric vectors with their directional compensation,

$$G_2 = \frac{V_A}{V} \left(2 - \frac{\left| \sum_{i=1}^{V_A} V_i \right|}{\sum_{i=1}^{V_A} |V_i|} \right), \tag{6}$$

where V is the total number of vectors, V_A is the number of remaining asymmetric vectors, and V_i is the i-th asymmetric vector. The first factor measures the fraction of preserved vectors, while the second quantifies their directional compensation. This formulation has also been employed in morphological applications of GPA, including galaxy characterization [Rosa et al. 2018].

The moments G_3 and G_4 complement the GPA formulation with properties related to gradient organization, including aspects of phase and complex representation [Rosa et al. 2000, Ramos et al. 2000, Rosa et al. 2003], but have not yet been implemented in the library.

3. GPA as a Software Library

GPA is provided as an open-source Python library with an object-oriented interface for parameter configuration and descriptor extraction, enabling users to select moments, configure tolerances, and obtain results for statistical or machine-learning analyses. It also provides visualizations of the gradient and asymmetric lattices and their Delaunay triangulation, as summarized in Fig. 1. Code and installation are available at: repository.

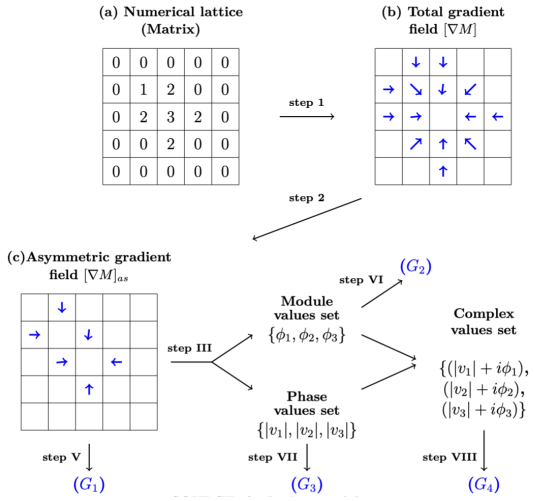


Figure 1. Computational workflow of the GPA library, from the input matrix to descriptor extraction and pattern visualization.

4. Applications and Demonstration

As a demonstration of the library, canonical spatial patterns with different levels of symmetry and organization are presented in Fig. 3. The symmetric pattern shows radially symmetric vector pairs that are removed, while the laminar pattern exhibits approximately uniform orientations and the asymmetric pattern retains most vectors. As shown in Fig. 2, both symmetric patterns tend to $G_1, G_2 \approx 0$ as the matrix size increases; for G_1, both laminar and asymmetric patterns tend to 2, whereas for G_2, the laminar pattern tends to 1 and the asymmetric pattern to 2. These trends are consistent with previous results.[1]

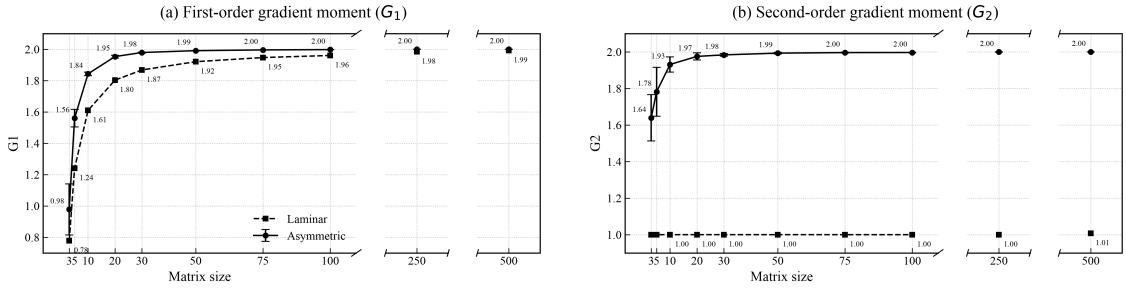


Figure 2. Gradient moments versus matrix size for asymmetric and laminar patterns (mean $\pm$ standard deviation for asymmetric values).

[1] See [Rosa et al. 1999], particularly Fig. 8 and Table 1 for G_1.

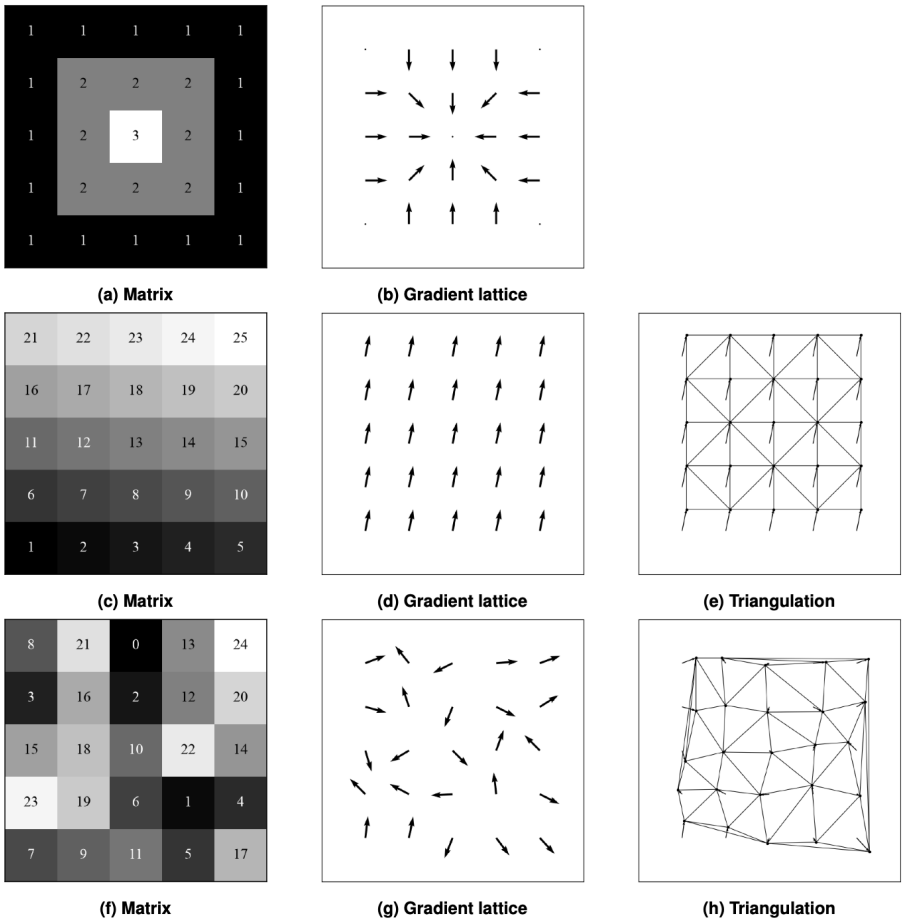


Figure 3. Application of the GPA library to canonical patterns. For each input, the original matrix and its corresponding gradient lattice are shown. For the asymmetric pattern, the resulting Delaunay triangulation is also presented.

5. Conclusion

In this work, we presented a preliminary Python library aimed at consolidating the GPA methodology into a unified, reproducible, and extensible computational framework. The current implementation integrates the main stages of the GPA workflow, including gradient construction, symmetry analysis, asymmetric gradient extraction, calculation of the first two gradient moments, G_1 and G_2, and visualization. The results obtained for canonical patterns reproduce the expected behavior of the current descriptors, demonstrating the potential of the library for quantitative characterization and feature extraction from spatial patterns. This work represents an initial stage in the consolidation of GPA into a standardized computational tool. Future developments will include the implementation of G_3 and G_4, systematic validation against previous results, performance evaluation, and extensions to additional types of data.

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