

An Abstract Data Type to Handle Vague Spatial Objects Based on the Fuzzy Model

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Abstract. *Crisp spatial data are geometric features with exact location on the extent and well-known boundaries. On the other hand, vague spatial data are characterized by inaccurate locations or uncertain boundaries. Despite the importance of vague spatial data in spatial applications, few related work indeed implement vague spatial data and they do not define abstract data types to enable the management of vague spatial data by using database management systems. In this sense, we propose the abstract data type FuzzyGeometry to handle vague spatial data based on the fuzzy model. FuzzyGeometry was developed as a PostgreSQL extension and its implementation is open source. It offers management for fuzzy points and fuzzy lines. As a result, spatial applications are able to access the PostgreSQL to handle vague spatial objects.*

1. Introduction

Spatial applications are commonly used for spatial analysis to aid in the decision making process. They mainly analyze spatial objects that can be represented by geometries [Güting 1994, Schneider and Behr 2006]. These geometries represent spatial objects of the real world, which can be simple, such as point, line, and polygon (i.e., region), or complex, such as multipoint, multiline, and multipolygon. Furthermore, these spatial objects have exact location in the extent, i.e., their geographical coordinates clearly define their geographical positions. In addition, a region has well-defined boundaries, expressing with exactness its limit. These objects are called *crisp spatial data*.

On the other hand, real-world phenomena frequently have inexact location, uncertain boundaries, or imprecise interior [Siqueira et al. 2014]. These objects are defined as *vague spatial data*. There are several models to represent vague spatial objects, such as *exact models* [Cohn and Gotts 1995, Pauly and Schneider 2008, Bejaoui et al. 2009, Pauly and Schneider 2010], *rough models* [Beaubouef et al. 2004], *probabilistic models* [Cheng et al. 2003, Li et al. 2007, Zinn et al. 2007], and *fuzzy models* [Dilo et al. 2007, Schneider 2008, Schneider 2014, Carniel et al. 2014]. These models discuss standards to represent vague spatial data as well as their operations. However, there are no native support of vague spatial data in Spatial Database Management Systems (SDBMS), such as PostgreSQL with the PostGIS extension. This is a problem since

spatial applications increasingly require the management of vague spatial objects in situations commonly found in real world, such as the representation of soil mapping, fire and risk zones, oceans, lakes, and air polluted areas.

To fill this gap, we propose an *abstract data type* (ADT) called FuzzyGeometry. FuzzyGeometry ADT is based on the fuzzy model and it is an open-source PostgreSQL extension. The fuzzy model represents vague spatial objects by making use of the fuzzy set theory [Zadeh 1965]. We use this model since we are able to represent different vagueness levels of an object. It is possible because each point has a value in the interval $[0, 1]$, called *membership degree*, which indicates the possibility of a point to belong to a spatial object. Vague spatial data modeled by using fuzzy models are designed as *fuzzy spatial data types*, such as fuzzy point, fuzzy line, and fuzzy region. In this paper, we focus on the design and implementation of fuzzy points and fuzzy lines only, which already have several challenges for their definition and implementations. In addition, we propose operations involving them, such as fuzzy geometric set operations.

This paper is organized as follows. Section 2 summarizes related work. Section 3 describes the technical background. Section 4 presents our FuzzyGeometry ADT. Section 5 concludes the paper and presents future work.

2. Related Work

Few works in the literature implement vague spatial data types based on the fuzzy model by using a database management system or a Geographic Information System (GIS). On the other hand, exact models are also frequently adopted to represent vague spatial objects since they use well-known crisp spatial algorithms. Hence, we compare implementations based on the exact model and the fuzzy model.

Despite there are several exact models [Cohn and Gotts 1995, Pauly and Schneider 2008, Bejaoui et al. 2009, Pauly and Schneider 2010], only few implementations are based on them. Vague Spatial Algebra (VASA) [Pauly and Schneider 2008, Pauly and Schneider 2010] is an exact model that offers several spatial operations for vague points, vague lines, and vague regions. An implementation is given in [Pauly and Schneider 2008]¹, which implements the VASA by adapting SQL functions to handle vague spatial objects. However, in this paper, we propose an abstract data type for vague spatial data based on the fuzzy model. While VASA has only three levels of representation for vague spatial objects (the crisp part, the vague part, and the part that certainly does not belong to the spatial object), vague spatial objects based on the fuzzy model may have several levels in the real interval $[0, 1]$. Therefore, it allows a more detailed representation of vague spatial data.

For vague spatial data based on the fuzzy model, the Spatial Plateau Algebra (SPA) [Schneider 2014] provides definitions for fuzzy spatial data that reuses crisp spatial algorithms. Hence, a fuzzy spatial object, called *spatial plateau object*, is defined as a finite sequence of pairs where each pair is formed by one crisp spatial object and a membership degree in $]0, 1]$. In addition, SPA defines spatial plateau operations to handle spatial plateau objects, such as geometric set operations. Though this implementation concept was proposed, spatial plateau data were not implemented in a SDBMS.

¹<http://www.cise.ufl.edu/research/SpaceTimeUncertainty/>

Vague spatial data based on the fuzzy model are implemented in [Kraipeerapun 2004, Dilo et al. 2006]. They implement the following vague spatial data types: vague point, vague line, and vague region. A vague point is defined as a tuple (x, y, λ) , where $(x, y) \in \mathbb{R}^2$ gives the location, and $\lambda \in]0, 1]$ gives the membership degree. A vague line is defined as a finite sequence of tuples $((x_1, y_1, \lambda_1), \dots, (x_n, y_n, \lambda_n))$ for some $n \in \mathbb{N}$, where each tuple is a vague point in the vague line constructed by using linear interpolation. A vague region is composed by several vague lines and the Delaunay triangulation, which is stored together with the vague region object. A membership degree of any point inside a vague region is calculated by using linear interpolation of membership degrees of vertices of the triangle to which the point belongs. This implementation is performed in the GRASS GIS, and not in a SDBMS. They do not implement the vague geometric difference between vague lines. However, our paper proposes an abstract data type implemented a SDMS (i.e., the PostgreSQL) to manage vague spatial objects.

3. Technical Background

This section summarizes the main needed concepts to understand our proposal of an ADT to handle vague spatial data. Section 3.1 summarizes vague spatial data concepts while Section 3.2 summarizes fuzzy set theory.

3.1. Vague Spatial Data

While crisp spatial objects have exact location and well-known boundaries, vague spatial objects have inexact location, uncertain boundaries, or imprecise interior. There are distinct models to represent vague spatial data that can be classified as *exact models* [Cohn and Gotts 1995, Pauly and Schneider 2008, Bejaoui et al. 2009, Pauly and Schneider 2010], *rough models* [Beaubouef et al. 2004], *probabilistic models* [Cheng et al. 2003, Li et al. 2007, Zinn et al. 2007], and *fuzzy models* [Dilo et al. 2007, Schneider 2008, Schneider 2014, Carniel et al. 2014].

Exact models aim to reuse existing abstract data types of crisp spatial data types (e.g. crisp points, crisp lines, and crisp regions) to represent vague spatial objects. In general, vague spatial objects are defined by using two crisp spatial objects. One object represents the vague spatial part while other object represents the well-known spatial part. The main relevant models are: *Egg-Yolk* [Cohn and Gotts 1995], *Qualitative Min-Max Model* (QMM) [Bejaoui et al. 2009], and *Vague Spatial Algebra* (VASA) [Pauly and Schneider 2008, Pauly and Schneider 2010]. Egg-Yolk model defines only vague regions, which are represented by two sub-regions: a sub-region denominates the yolk (i.e., vague spatial part) and other sub-region denominates the egg (i.e., well-known spatial part) that is contained in the yolk part. QMM model defines vague spatial objects by using two limits, a minimum limit (i.e., well-known spatial part) and a maximum limit (i.e., includes the minimum limit and extends to the part that possibly belongs to the spatial object). In addition, this model uses qualitative classifications of vagueness levels, such as *completely crisp*, *partially vague*, and *completely vague*. Finally, VASA defines a vague spatial object as a pair of disjoint or adjacent crisp spatial objects of the same type.

Rough models are based on the rough set theory [Pawlak 1982] that defines a lower and an upper approximation. Hence, a vague spatial object is represented by these

approximations. Lower spatial approximation of an object is a subset of its upper spatial approximation. Vague spatial data represented by rough models deal with vague spatial objects with inexact location as well as inexact measures [Beaubouef et al. 2004].

Probabilistic models are based on the probability density functions [Cheng et al. 2003, Li et al. 2007, Zinn et al. 2007] and the treatment of spatial vagueness is performed through objects positions and measures. In general, these models handle with expectative of a future event based on the known-characteristics. While the probability density functions are exacts, the location of an object is uncertain.

Fuzzy models, that is, models based on the fuzzy set theory [Zadeh 1965], assign membership degrees in $[0, 1]$ for each point of the location to represent spatial vagueness in different levels. There are several representations of vague spatial data by using the fuzzy set theory. Fuzzy Minimum Boundary Rectangle [Somodevilla and Petry 2004] includes the fuzzy set theory in order to define membership functions by using several Minimum Boundary Rectangles. Fuzzy spatial data types denominate fuzzy points, fuzzy lines, and fuzzy regions as well as fuzzy geometric set operations, such as, fuzzy geometric union, fuzzy geometric intersection, and fuzzy geometric difference has been defined [Dilo et al. 2007, Schneider 2008, Schneider 2014, Carniel et al. 2014]. In addition, vague partitions and their operations are defined [Dilo et al. 2007]. In this paper, we considered the fuzzy spatial data defined in [Dilo et al. 2007] as design goals to implement the FuzzyGeometry. Section 3.2 summarizes needed fuzzy set theory concepts.

3.2. Fuzzy Set Theory

Fuzzy set theory [Zadeh 1965] is an extension and generalization of the classic (crisp) set theory. In the classic theory, let X be a crisp set of objects, called the *universe*. The subset A of X can be described by a function $\chi_A : X \rightarrow \{0, 1\}$, which for all $x \in X$, $\chi_A(x)$ is 1 if and only if, $x \in A$ and 0 otherwise. On the other hand, fuzzy set theory defines a function $\mu_{\tilde{A}}$ that maps all elements of X in the real interval $[0, 1]$ by assigning *membership degrees* in a specific set. Hence, fuzzy set theory allows that an element x has different membership values in different fuzzy sets. Let X be the universe. Then, the function $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is called of *membership function of the fuzzy set \tilde{A}* . Therefore, each element of fuzzy set \tilde{A} has a membership degree in the real interval $[0, 1]$ according to the membership function: $\tilde{A} = \{(x, \mu_{\tilde{A}}) \mid x \in X\}$.

Classic operations among crisp sets also are extended for the fuzzy sets. We will summarize these operations. Let \tilde{A} and \tilde{B} be fuzzy sets in X , then the intersection, union, and difference are defined as follows, respectively:

- $\tilde{A} \cap \tilde{B} = \{(x, \mu_{\tilde{A} \cap \tilde{B}}(x)) \mid x \in X \wedge \mu_{\tilde{A} \cap \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))\}$
- $\tilde{A} \cup \tilde{B} = \{(x, \mu_{\tilde{A} \cup \tilde{B}}(x)) \mid x \in X \wedge \mu_{\tilde{A} \cup \tilde{B}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))\}$
- $\tilde{A} - \tilde{B} = \{(x, \mu_{\tilde{A} - \tilde{B}}(x)) \mid \mu_{\tilde{A} - \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), 1 - \mu_{\tilde{B}}(x))\}$

An alpha-cut (α -cut) and a strict alpha-cut (strict α -cut) of the fuzzy set \tilde{A} for a specific value α is a crisp set defined as follows, respectively:

- $\tilde{A}^{\geq \alpha} = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \wedge 0 \leq \alpha \leq 1\}$
- $\tilde{A}^{> \alpha} = \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha \wedge 0 \leq \alpha < 1\}$

When α value is 1 for the α -cut of \tilde{A} , the result is called of core of \tilde{A} .

Generalizations of fuzzy sets operations, such as intersection and union, replace the min and max operators by *triangular norms* (t-norm) and *triangular co-norms* (s-norm), respectively. A t-norm T is defined as a commutative, associative, non-decreasing binary operation on $[1, 0]$, with signature $T : [0, 1]^2 \rightarrow [0, 1]$ satisfying the following boundary conditions, $T(1, x) = x$ and $T(0, x) = 0$ for all $x \in [0, 1]$ [Klement et al. 2000]. Let $x, y \in [0, 1]$, some examples of t-norms are listed as follows:

- $T^*(x, y) = \begin{cases} x & \text{if } y = 1 \\ y & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$ (*drastic intersection*)
- $T_p(x, y) = ab$ (*product t-norm*)
- $T_l(x, y) = \max(0, x + y - 1)$ (*Lukasiewicz t-norm*)

For any t-norm there is an s-norm, which is obtained by De Morgan's laws. Hence, a s-norm is defined as a commutative, associative, non-decreasing binary operation on $[1, 0]$, with signature $S : [0, 1]^2 \rightarrow [0, 1]$ satisfying the following boundary conditions, $S(1, x) = 1$ and $S(0, x) = x$ for all $x \in [0, 1]$ [Klement et al. 2000]. Let $x, y \in [0, 1]$, some examples of s-norms are listed as follows:

- $S^*(x, y) = \begin{cases} x & \text{if } y = 0 \\ y & \text{if } x = 0 \\ 1 & \text{otherwise} \end{cases}$ (*drastic union*)
- $S_p(x, y) = x + y - xy$ (*probabilistic sum*)
- $S_l(x, y) = \min(1, x + y)$ (*bounded sum*)

The height of a fuzzy set \tilde{A} is defined as the greatest membership value (sup) of the membership function of \tilde{A} [Jamshidi et al. 1993], that is, $h(\tilde{A}) = \sup_x [\mu_{\tilde{A}}(x)]$. A fuzzy set \tilde{A} is called normal when $h(\tilde{A}) = 1$, and subnormal when $h(\tilde{A}) < 1$. To normalize a fuzzy set \tilde{A} , we apply the normalization, which is defined as $Norm_{\mu_{\tilde{A}}}(x) = [\mu_{\tilde{A}}(x)/h(\tilde{A})]$ for all $x \in X$.

The concentration (CON) of a fuzzy set \tilde{A} decreases the fuzziness, while the dilation (DIL) of a fuzzy set \tilde{A} increases the fuzziness [Jamshidi et al. 1993]. They are defined as follows:

- $\mu_{CON(\tilde{A})}(x) = [\mu_{\tilde{A}}(x)]^p$ for all $x \in X$ where $p > 1$
- $\mu_{DIL(\tilde{A})}(x) = [\mu_{\tilde{A}}(x)]^r$ for all $x \in X$ where $r \in]0, 1[$

Finally, there are some notations to textually represent a fuzzy set [Jamshidi et al. 1993]. The following definitions are textual representation of a fuzzy set \tilde{A} :

- $\tilde{A} = \sum_{x_i \in X} \mu_{\tilde{A}}(x_i)/x_i$ when X is finite and discrete
- $\tilde{A} = \int_x \mu_{\tilde{A}}(x)/x$ when X is continuous

Note that the signs of sum and integral denote the union of the membership degrees and the slash (/) denotes a separator.

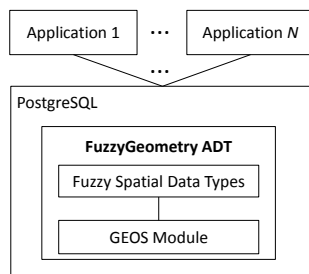


Figure 1. Architecture of the FuzzyGeometry ADT.

4. The FuzzyGeometry Abstract Data Type

We propose a novel ADT to handle vague spatial data based on the fuzzy model (Section 3.1), called FuzzyGeometry. We implemented the FuzzyGeometry ADT as a PostgreSQL extension. PostgreSQL has free license and it is an extensible database management system, which new ADTs can be implemented by using a low level program language (e.g., C language) or a high level program language (e.g. pl/pgSQL). FuzzyGeometry was implemented in the C language by using the extensibility provided by the PostgreSQL internal library.

Figure 1 shows the architecture of the FuzzyGeometry ADT in the PostgreSQL. Our ADT uses the GEOS module for crisp geometric operations that were adapted for vague spatial data based on the fuzzy model. As discussed in the Section 2, related work try to use crisp geometric set operations to handle vague spatial data. Therefore, we use the GEOS module for this purpose. GEOS module is a library in C/C++ with free source code used to handle crisp spatial data in GIS, such as the GRASS, and SDBMS, such as the PostGIS extension of PostgreSQL. As a result, external applications can use the FuzzyGeometry ADT by accessing directly the PostgreSQL.

In the next sections we will detail the FuzzyGeometry ADT. Section 4.1 presents the FuzzyGeometry data types and their textual representations. Section 4.2 details operations for each data type of the FuzzyGeometry.

4.1. Fuzzy Spatial Data Types of the FuzzyGeometry

The FuzzyGeometry ADT offers the following fuzzy spatial data types (Figure 1): fuzzy points and fuzzy lines. They can be simple or complex, and the hierarchy among them is showed in Figure 2. The highest level of hierarchy is the *FuzzyGeometry data type*. A FuzzyGeometry can be a *Simple FuzzyGeometry* or a *Complex FuzzyGeometry*. The Simple FuzzyGeometry data type cannot be instanced since it is a generalization for simple fuzzy spatial objects, which can be instanced as a fuzzy point and a fuzzy linestring (i.e., a fuzzy line). Similarly, the Complex FuzzyGeometry data type cannot be instanced since it is a generalization for complex fuzzy spatial objects that are collections of simple fuzzy spatial objects of the same type, which can be a fuzzy multipoint (i.e., a complex fuzzy point) and a fuzzy multilinestring (i.e., a complex fuzzy line). Hence, Figure 2 shows in gray, the possible data types that can be instanced. Note that the current version of the FuzzyGeometry ADT does not support for fuzzy regions.

These fuzzy spatial objects have a membership degree for each point in the space

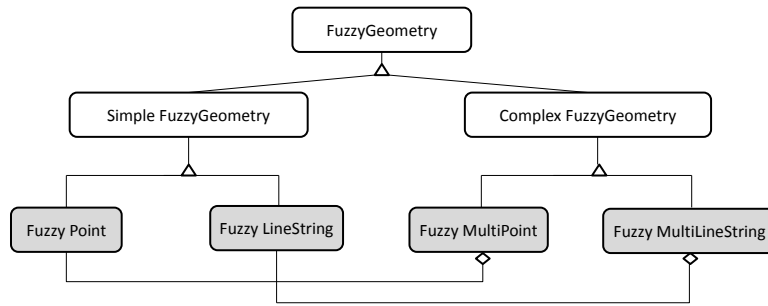


Figure 2. The hierarchy of the FuzzyGeometry data types.

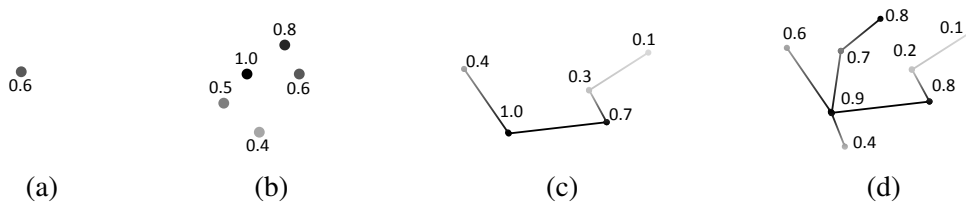


Figure 3. Examples of fuzzy spatial objects of the FuzzyGeometry: (a) a fuzzy point object, (b) a fuzzy multipoint object, (c) a fuzzy linestring object, and (d) a fuzzy multilinestring object. Darker parts have higher membership values than lighter areas.

to denote inexact location or imprecision. Such membership degree assigns the value in the real interval $[0, 1]$ for each point, which determines the spatial vagueness in a point. We will detail each possible instances of a FuzzyGeometry object according to Figure 2.

A fuzzy point is defined as (x, y, u) , which (x, y) corresponds to a coordinate pair that provides the location and u is the membership degree in the interval real $]0, 1]$ for its coordinate pair. A set of disjoint fuzzy points form a fuzzy multipoint. Hence, a fuzzy multipoint is defined as a sequence of triples in the format (x, y, u) . An unique fuzzy point is a special case for a fuzzy multipoint. Figure 3a shows an example of a fuzzy point object and Figure 3b shows an example of a fuzzy multipoint object.

A fuzzy linestring is defined as a sequence $((x_1, y_1, u_1), \dots, (x_n, y_n, u_n))$ for some $n \in \mathbb{N}$, i.e., a sequence of fuzzy points. These fuzzy points are sequentially linked. For instance, (x_1, y_1, u_1) and (x_2, y_2, u_2) are two linked points and thus, they form a segment line. A membership degree of a given point in the fuzzy linestring is calculated by using linear interpolation on a segment. A fuzzy linestring object should not has self-intersection. A fuzzy multilinestring is defined as a set of fuzzy lines that can only intersect in their endpoints. An unique fuzzy linestring is a special case for a fuzzy multilinestring. Figure 3c shows an example of a fuzzy linestring object and Figure 3d shows an example of a fuzzy multilinestring object composed by 4 fuzzy linestring objects.

In the PostgreSQL, only the FuzzyGeometry data type is specified, which can be internally instanced as a fuzzy point, fuzzy linestring, fuzzy multipoint, and fuzzy multilinestring. To handle these data types by using the SQL language, we define input and output functions. An input operation transforms textual representation into an internal

representation; An output operation transforms the internal representation into the textual representation. Hence, in order to insert fuzzy spatial objects in relational tables in the PostgreSQL, we propose textual representations for simple and complex fuzzy points and lines. Therefore, by using these representations, users are able to insert fuzzy spatial objects into tables and to visualize them as results of spatial queries.

We define textual representations of fuzzy spatial data types based on the textual representations of fuzzy sets (Section 3.2). In general, firstly appears the name of fuzzy spatial data type and in parentheses, for each point, its membership degree and its coordinate pairs separated by slash. Empty fuzzy spatial objects, which contain no coordinates and membership values, can be specified by using the symbol EMPTY after the data type name. Let (x, y) be a coordinate pair, u be a membership degree in real interval $]0, 1]$, and $k, j \in \mathbb{N}$. Then, we define the textual representation for fuzzy point (i), fuzzy multipoint (ii), fuzzy linestring (iii), and fuzzy multilinestring (iv) as follows:

- (i) FUZZYPOINT($u/x\ y$)
- (ii) FUZZYMULTIPOINT($u_1/x_1\ y_1, \dots, u_k/x_k\ y_k$)
- (iii) FUZZYLINESTRING($u_1/x_1\ y_1, \dots, u_k/x_k\ y_k$)
- (iv) FUZZYMULTILINESTRING($((u_{1_1}/x_{1_1}\ y_{1_1}, \dots, u_{k_1}/x_{k_1}\ y_{k_1})_1, \dots, (u_{1_j}/x_{1_j}\ y_{1_j}, \dots, u_{k_j}/x_{k_j}\ y_{k_j})_j)$)

4.2. FuzzyGeometry Operations

In this paper, we consider the grouping provided in [Güting 1994] for abstract models of spatial data to classify and define the FuzzyGeometry operations. The groups are:

- (i) Operations that return spatial objects. For instance, geometric set operations.
- (ii) Operations that return topological relationship between spatial objects. For instance, the topological relationships between two lines.
- (iii) Operations that return numbers. For instance, metric operators.
- (iv) Operations on set of objects. For instance, spatial aggregate functions, such as the geometric union on a set of objects.

FuzzyGeometry implements operations of groups (i) and (iii). In the following sections, we will show these operations according to their classifications.

4.3. Geometric Set Operations

Geometric set operations of the FuzzyGeometry ADT are: union, intersection, and difference. In general, we used the formal definition provided by fuzzy models that define vague spatial data (Section 3.1) to implement them. These geometric set operations belong to group (i), i.e., operations that return spatial data. Let $\tilde{A}, \tilde{B}, \tilde{C}$ be FuzzyGeometry objects, $s \in \{\max\ s\text{-norm}, \text{drastic } s\text{-norm}, \text{probabilistic } s\text{-norm}, \text{bounded } s\text{-norm}\}$, $t \in \{\min\ t\text{-norm}, \text{drastic } t\text{-norm}, \text{product } t\text{-norm}, \text{Lukasiewicz } t\text{-norm}\}$, and $d \in \{\text{fuzzy difference}, \text{arithmetic difference}\}$. Then, we define the following signatures (the prefix $FG_$ is used in all operations of the FuzzyGeometry ADT):

- (i) $FG_Union(\tilde{A}, \tilde{A}, s) \rightarrow \tilde{A}$
- (ii) $FG_Union(\text{set of } \tilde{A}) \rightarrow \tilde{A}$
- (iii) $FG_Intersection(\tilde{A}, \tilde{B}, t) \rightarrow \tilde{C}$
- (iv) $FG_Difference(\tilde{A}, \tilde{A}, d) \rightarrow \tilde{A}$

The union operation (i) between FuzzyGeometry objects is performed by the spatial union and the fuzzy union of the intersecting points. The fuzzy union can be executed by using a specific s-norm s . In addition, the union is only executed for FuzzyGeometry objects of same type. For instance, the union of two fuzzy linestrings yields other fuzzy linestring, which its location is the spatial union and its membership degrees for each point are calculated using the s-norm s . Other s-norms can be implemented as well. In addition, the union operation can be used as an aggregation operator, i.e., the operation (ii). This means that, given a set of FuzzyGeometry objects, this operation yields the union among all the objects contained in this set. The strategy to compute this aggregation is to perform the union operation (i) incrementally for each FuzzyGeometry object contained in the set by considering a default s-norm (i.e., the max s-norm).

The intersection operation (iii) between FuzzyGeometry objects is performed by the spatial intersection and the fuzzy intersection of the intersecting points. The fuzzy intersection can be executed by using a specific t-norm t . The intersection can be executed between FuzzyGeometry objects of different types, and the resulting FuzzyGeometry object is the lower data type by considering the hierarchy: *fuzzy linestring* > *fuzzy point*. For instance, the intersection between fuzzy linestrings and fuzzy points yields a fuzzy point or a fuzzy multipoint object composed by the commons points with membership degree calculated by using a t-norm t . Other t-norms can be implemented as well.

The difference operation (iv) between FuzzyGeometry objects is performed by the spatial difference and the membership degrees of the intersecting points are calculated by using a difference operator. The membership degrees are calculated by using fuzzy difference or arithmetic difference. The arithmetic difference is defined by $diff(a, b) = a - b$ if $a > b$; 0 otherwise. For instance, the difference between fuzzy linestrings yields a fuzzy linestring, where commons locations will have the membership degree calculated by the fuzzy difference or arithmetic difference, and the spatial difference is performed for the remaining locations.

4.4. Generic Operations

Generic operations of the FuzzyGeometry ADT are: core, boundary, set linguistic term, and crisp transform. These operations can be applied in any type of FuzzyGeometry object and belong to group (i) (operations that return spatial data). Let \tilde{A} be a FuzzyGeometry object, B be a crisp spatial object, and lt be a linguistic term. Then, we define the following signatures:

- (i) $FG_Core(\tilde{A}) \rightarrow \tilde{A}$
- (ii) $FG_Boundary(\tilde{A}) \rightarrow \tilde{A}$
- (iii) $FG_Set_LinguisticTerm(\tilde{A}, lt) \rightarrow \tilde{A}$
- (iv) $FG_CrispTransformation(\tilde{A}) \rightarrow B$

The core (i) and boundary (ii) operations get the locations that have exact locations (locations with membership degree equal to 1) and vague locations (locations with membership degree less than 1 and greater than 0), respectively.

In this paper, we propose that fuzzy spatial objects (i.e., FuzzyGeometry objects) can have linguistic terms since it represents a specific vague spatial phenomenon. For instance, a fuzzy linestring object represents a determinate phenomenon. This phenomenon

can have characteristics that identify itself. An example is showed as follows. An attribute stores fuzzy linestrings that represent animal routes. Animal routes can have linguistic terms that symbolize the frequency that an animal appears in a local. For instance, *all the time*, *sometimes*, and *few times*. Hence, a fuzzy linestring object has a linguistic term associated that indicate the frequency of such animal route. For instance, an animal route R that has as linguistic term *sometimes*. This means that each point of the fuzzy linestring object \tilde{A} that represents the animal route R has a membership degree to indicate the level of a frequency, which is *sometimes*. Therefore, let p be a point of \tilde{A} with membership degree equal to 0.8, then it indicates 80% of chance to an animal appears *sometimes* at point p . Linguistic terms are commonly used in the fuzzy logic. To do this operation, we propose the operation (iii).

The operation (iv) transforms FuzzyGeometry objects into crisp spatial objects (i.e., in PostGIS objects). This means that, the membership degrees disappear. That is, the resulting crisp spatial object is formed by all the points with membership degree greater than 0.

4.5. Operations Based on the Fuzzy Set Theory

The operations based on the fuzzy set theory of the FuzzyGeometry ADT are: fuzzy spatial alpha-cut, fuzzy spatial strict alpha-cut, fuzzy spatial concentration, fuzzy spatial dilation, fuzzy spatial height, and fuzzy spatial normalization. These operations are adaptations of fuzzy operations (Section 3.2) to deal with fuzzy spatial objects. The fuzzy spatial alpha-cut, fuzzy spatial strict alpha-cut, fuzzy spatial concentration, fuzzy spatial dilation, and fuzzy spatial normalization operations belong to group (i) (operations that return spatial data). The fuzzy spatial height belongs to group (iii) (operations that return numbers). Let \tilde{A} be a FuzzyGeometry object, $\alpha \in [0, 1]$, $p > 1$, $r \in]0, 1[$, and $h \in]0, 1]$. Then, we define the following signatures:

- (i) $FG_Alphacut(\tilde{A}, \alpha) \rightarrow \tilde{A}$
- (ii) $FG_StrictAlphacut(\tilde{A}, \alpha) \rightarrow \tilde{A}$
- (iii) $FG_Concentration(\tilde{A}, p) \rightarrow \tilde{A}$
- (iv) $FG_Dilation(\tilde{A}, r) \rightarrow \tilde{A}$
- (v) $FG_Height(\tilde{A}) \rightarrow h$
- (vi) $FG_Normalization(\tilde{A}) \rightarrow B$

The fuzzy spatial alpha-cut (i) and the fuzzy spatial strict alpha-cut (ii) operations filter the locations that have membership degree equal to or greater or equal to α , respectively. These operations are useful to identify locations that contain specifics membership degrees.

The fuzzy spatial concentration (iii) and dilation (iv) operations decrease and increase the membership degrees of the locations, respectively. These operations are useful to analyze or edit locations in order to intensify or smooth the spatial vagueness. In addition, these operations can change the meaning of linguistic term associated with the vague spatial object.

The fuzzy spatial height (v) operation returns the fuzzy height of the membership degrees of a FuzzyGeometry object. This operation is necessary for the fuzzy spatial normalization (vi) operation. If the height of a FuzzyGeometry object is 1, than it has a core. Otherwise, the normalization (vi) can be used to enforce an existence of a core, which will be the locations with the higher membership degree.

5. Conclusions and Future Work

In this paper, we proposed a novel abstract data type called FuzzyGeometry to handle vague spatial objects based on the fuzzy model in the PostgreSQL. Vague spatial data are an important representation of real-world phenomena that have vague characteristics, i.e., inexact location or uncertain boundaries. Fuzzy model can be adequately used for representation of spatial vagueness. Although this model was proposed, implementations are limited and have not been incorporated into SDBMS. Hence, FuzzyGeometry ADT advances in the state of art to handle vague spatial objects in a SDBMS.

Several operations have been proposed and implemented to handle FuzzyGeometry objects. Among them is the implementation of geometric set operations. Additionally, we proposed the use of linguistic terms to characterize a vague spatial object. Further, we propose textual representations in order to insert and retrieve FuzzyGeometry objects in a spatial database. Finally, we propose new operations based on the fuzzy set theory, such as fuzzy spatial concentration and fuzzy spatial dilation.

Future work will deal with the topological relationship between vague spatial objects based on the fuzzy model, such as the implementation of the predicate “overlap” [Carniel et al. 2014]. Further, the implementation of the fuzzy region data type. We will also propose algorithms in order to extract information by using fuzzy inference [Jamshidi et al. 1993] on vague spatial objects based on the fuzzy model.

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